A Monstrous Inference called Mahāvidyānumāna and Cantor’s Diagonal Argument

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Abstract A mahāvidyā inference is used for establishing another inference. Its Reason (hetu) is normally an omnipresent (kevalānvayin) property. Its Target (sādhyā) is defined in terms of a general feature that is satisfied by different properties in different cases. It assumes that there is no (relevant) case that has the absence of its Target. The main defect of a mahāvidyā inference μ is a counterbalancing inference (satpratipakṣa) that can be formed by a little modification of μ. The discovery of its counterbalancing inference can invalidate such an inference. This paper will argue that Cantor’s diagonal argument too shares some features of the mahāvidyā inference. A diagonal argument has a counterbalanced statement. Its main defect is its counterbalancing inference. Apart from presenting an epistemological perspective that explains the disquiet over Cantor’s proof, this paper would show that both the mahāvidyā and diagonal argument formally contain their own invalidators.

Keywords Mahāvidyā · Kulārka Paṇḍita · Mahādeva Bhaṭṭavāḍindra · Counterbalancing · Cantor · Diagonal argument

In the first half of this paper, I shall discuss the features of an all-proving inference, namely the mahāvidyā inference, and its defects. In the thirteenth century, some Indian epistemologists came up with an ingenious epistemological framework that could detect the defects of the mahāvidyā inference and invalidate it. After the invention of that anti-mahāvidyā framework, the mahāvidyā inference ceased to be
used. In the second half of the paper, I shall use that framework to account for the intuitive discomfort about Cantor’s diagonal argument.

Let us first briefly discuss anumāṇa or inference. Suppose it has been known that $B$ exists in every observed locus of $A$. $A$ does not exist in any locus which $B$ is absent from. After having known all these, when the subject finds $A$ in a locus $S$, they infer, ‘$S$ has $B$, since it has $A$’. Here $B$, which is inferred from $A$, is the sādhya or Target, $A$ is the hetu or Reason and the locus $S$, where the Target is inferred, is the pākṣa or Site. The loci that are known to have the Target are called sapakṣas or Co-sites. It is evident that at least some of the Co-sites must have the Reason. The relation between the Target and Reason is ascertained through their co-presence in some Co-sites, and those Co-sites serve as the ‘base cases’ (dṛṣṭāntas) for the inference. We may note here that all base cases are Co-sites while the reverse is not necessarily true. On the contrary, each of the loci that are known to have the absence of the Target must have the absence of the Reason too. Such loci are called vipakṣas or Anti-sites. The following event exemplifies the inferential method. One has observed that all the (observed) cases of smoke are cases of fire. The loci of fire are kitchens, sacrificial altars and blacksmith shops. Thus these are the Co-sites. The Co-sites that possess both fire and smoke are the ‘base cases’ of a possible inference. In this case, those Co-sites are kitchens and sacrificial altars. In a blacksmith shop, fire exists without being accompanied by smoke, since a red-hot iron-ball does not emit smoke. Smoke exists in many loci of fire and does not exist in any locus, which fire is absent from. Table 1 may represent the whole picture.

With this knowledge, when one sees smoke on a hill, one infers fire there. Here the hill, smoke and fire are the Site, Reason and Target respectively. The loci that are known to have fire are Co-sites, and the loci that are known to have the absence of fire are Anti-sites. The key here is the knowledge that every locus of smoke is a locus of fire. It is called the knowledge of pervasion or vyāpti-jñāna, which is ascertained through the observation of the co-presence of smoke and fire in base cases, and the observation of their co-located absences in the Anti-sites. After attaining this knowledge, the subject would definitely infer the Target wherever the Reason is found, unless there is some strong reason or evidential ground for denying

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Table 1  Ascertainment of Pervasion.

<table>
<thead>
<tr>
<th>Anti-site</th>
<th>Co-site</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-T, -R$</td>
<td>$+T$</td>
</tr>
<tr>
<td>Base Case</td>
<td></td>
</tr>
<tr>
<td>$+T, +R$</td>
<td>$+T, -R$</td>
</tr>
</tbody>
</table>

$T$ Target, $R$ Reason, $+X$ $X$ is present, $-X$ $X$ is absent.

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1 Matilal translates sapakṣa and vipakṣa as ‘homologue’ and ‘heterologue’. For these terms and their definitions, see Matilal (1998, p. 6).
the relation of pervasion. The inference is valid by default. But not finding a counterexample (i.e., the presence of the alleged Reason unaccompanied by the alleged Target) does not mean that there is none. Still, unavailability of the counterexample licenses an inference. Mahāvidyā inferences take advantage of this weakness of the inferential process and try to establish what is not supposed to be established otherwise.

A Brief History of Mahāvidyā Inferences

Very little is known about the origin of the mahāvidyā inference. We know why it was invented. I quote a translation of two verses, authored by Bhuvanasundarasūri (fifteenth century), from Telang (1920, p. viii):

The Bhaṭṭas (followers of the Mīmāṃsā school of Kumārila Bhaṭṭa) hold sound to be eternal but the Yaugas i.e. Vaiśeṣikas or Naiyāyikas hold it to be non-eternal. Hence a controversy arose between them. Therefore in order to convince the Bhaṭṭa disputants of the non-eternity of sound the great Ācārya of the Yaugas created the Mahāvidyā syllogisms.

Yauga means the Nyāya-Vaiśeṣika syncretism. But who is this great Ācārya of the syncretic school? There are two possibilities.

1. Kulārka Pandita authored a monograph entitled Daśaślokīṃmahāvidyāsūṭram (henceforth DM) in the eleventh century A.D. In that, he gives in the mysteries of composing the mahāvidyānumāna, a monstrous inference ‘that could prove anything one desired to prove’, svabhimatā-sakala-prameya-sādhaka.3 Kulārka was a follower of the Nyāya-Vaiśeṣika tradition (yaugācārya).

2. In his Mahāvidyāvidambanam (MVD), Mahādeva Bhaṭṭavaḍindra (thirteenth century) said that he wrote MVD in order to refute the theory of Mahāvidyā pioneered by logicians like Śivāditya (Miśra).4 Śivāditya (11/12 century) was one of the founders of the Nyāya-Vaiśeṣika syncretism. In his Lakṣaṇamālā, he introduced a few definitions that were called mahāvidyā-lakṣaṇas. Those definitions are based on a logical trick very similar to the trick that underlies the mahāvidyā inferences. In Pratyaktattvapradīpikā, Citsukha discusses and criticizes those.5

It is not easy to identify yaugācārya. I personally think that both Kulārka and Śivāditya knew the mahāvidyā technique that preexisted them. Kulārka used it for making inferences whereas Śivāditya used it for making definitions.6 Mahādeva mainly targeted the mahāvidyā inference-patterns discussed by Kulārka. An

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2 See the entry on Mahāvidyāvidambana in Potter (1977, p. 647).
3 Mahāvidyāvidambanam by Mahādeva, Telang (1920, p. 149).
4 Telang (1920, p. v).
5 Giri (1993, pp. 270, 273).
6 Matilal (1977, p. 101) was perhaps wrong in saying that ‘some Vedāntins’ introduced ‘such a mind-boggling inference pattern called “Mahāvidyā”’. 
unknown author wrote a commentary entitled *Mahāvidyāvivaranam (MV)* on Kulārka’s text.

Several philosophers including Amalānanda (thirteenth century), the author of *Vedāntakalpataru*, used Mahāvidyā inferences for silencing their opponents.\(^7\) The inferences were very successful until Mahādeva wrote his *Mahāvidyāvidambanam (MVD)*. According to Telang (1920, p. xi), Mahādeva’s epithets ‘viz. Nyāyāchārya, Parama-pandita and Bhatta signify that he was a great logician, Mīmāṃsaka, and well-versed in the Śāstras’. *Mahāvidyāvivaraṇaṭīppanam (MVT)* is Bhuvanasundarāśuri’s commentary on *MV*. Bhuvanasundara wrote a commentary on *MVD* (namely *Mahāvidyāvidambanavṛtti (MVDV)*) too and summarized the entire content of *MVD* in his monograph *Laghumahāvidyāvidambanam (LMV)*. Telang (1920, p. x) writes:

> Many books were written in support as well as in refutation of Mahāvidyā. But none of them seemed to have lived after Vādindra wrote his Mahāvidyā-vidambana and completely exploded the theory of Mahāvidyā syllogisms by proving them to be fallacious. Hence none of the authors after the fifteenth century refers to these syllogisms.

The entry on *Mahāvidyāvidambana* (summary by E. R. Sreekrishna Sarma) in Potter (1977) presents a very short summary of *MVD* in English. I would not have been able to understand the features of Mahāvidyā without reading *MV* and *MVT*. But I do not completely agree with them. In this paper, I shall partially follow their line of interpretation.

### The Structure and Applications of Mahāvidyā Inferences

Mahāvidyā inferences are peculiar. They are very cleverly crafted cheating devices. They actually work for other inferences. Those main inferences (*mūlānumānas*) are the real concern of the debater. The Site, Co-site etc. of those main inferences will be our main Site, main Co-site etc. respectively. Thus, just ‘Site’ etc. will refer to the terms of the associated mahāvidyā inference. Suppose the debater wants to establish that *X* is *Y* since *Z*, and they do not have a direct way for doing that. In that case, they will invoke a mahāvidyā inference, ‘*A* is *B*, since *C*’. *C* will normally be a property that resides in all the entities (*kevalānvayi*). In order to avoid confusions, we shall call *X*, *Y* and *Z*, i.e., the terms of the main inference, the main Site, main Target and main Reason. *A*, *B* and *C*, i.e., the terms of the mahāvidyā inference, will be our Site, Target and Reason. The main Site and main Target will definitely figure in the Site or Target.

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\(^7\) Śāstrī and Pansikar (1982, p. 567).

\(^8\) In *DM*, Kulārka gives us sixteen mahāvidyā schemata.
We shall discuss the general features of mahāvidyā inferences after considering a few schemata or formal patterns for constructing a mahāvidyā.8

Assumptions

The assumptions shared by both the sides of the debate are:

A1. Pots, clothes etc. are non-eternal (anitya).
A2. The sky (ākāśa) and souls are eternal (nitya).
A3. Every entity is knowable (meya).
A4. Not being present in both A and B means one of the three options: (i) being absent from A only, (ii) being absent from B only, and (iii) being absent from A and being absent from B. Thus not being present in both A and B is equivalent to being absent from at least one of them.9

Note: In MVT, Bhuvanasundara writes:

Something that is absent from both the eternal and non-eternal entities is not present in both. Something that is absent from the eternal entities only is not present in both. Something that is absent from the non-eternal entities only is also not present in both.10

A5. Pots, cloths etc. are created by sentient beings (kartṛ).
A6. The sky, souls, space and time are not created by anybody.

Symbols

In this paper, I shall use just three truth-functional operators:

~ = It is not the case that = Negation
∨ = or = Disjunction
∙ = and = Conjunction

Other symbols are:

= = Identity
≡ = Equivalence

Schema 1

apakṣasādhyavadvṛtti vipakṣānvayi yan na tat, sādhyavadvṛttitāyuktām sādhyate sādhyavarjite. (ātmā sabdētarāṇityaṇityavṛttitvānadhikaraṇāṇityaṇityavṛttidharmavān, meyatvāt, ghaṭavat.) Verse 1, DM11

9 This is an ancient version of the following rule of De Morgan: \( \sim (p \cdot q) \equiv \sim p \lor \sim q \).
10 Telang (1920, p. 658).
11 Telang (1920, p. 155).
Translation: $x$, which is not present in both the main Co-site (i.e., that, which possesses the Target without being the Site = *apakṣa-sādhya-vat*) and main Anti-site and which is present in an entity that possesses the main Target, gets established in the main Anti-site. E.g., the soul has a property $P$ such that (i) $P$ does not reside in both eternal and non-eternal entities other than sound, and (ii) $P$ resides in a non-eternal entity, since the soul is knowable, like a pot.

Explanation: The main inference is: Sound is non-eternal, since $γ$. The main Reason $γ$ is not our concern. It is not even relevant. The job of a Reason is to establish a Target. But here the responsibility of the main Reason has been completely handed over to the dummy Reason, i.e., the Reason of its associated *mahāvidyā* inference. After the dummy Reason establishes the *mahāvidyā* inference, the latter functions as $γ$ and establishes the main inference. We shall try to understand this by analyzing the examples.

The known loci of non-eternity, i.e., the main Co-sites are the pots, clothes etc. The known loci of the absence of non-eternity, i.e., the main Anti-sites are the sky, souls etc. since they are eternal.

Now we shall turn to the *mahāvidyā* inference. Both the Site and Co-site are supposed to have the Target. If something has the Target (*sādhya-vat*) without being the Site (*apakṣa*), it is a Co-site. Thus an *apakṣa-sādhya-vat* is a Co-site. The main Co-sites are the loci of non-eternity, i.e., pots, clothes etc. (i) We shall consider a property $P$ that will not be present in both the non-eternal and eternal entities, other than sound. We do not know whether sound is eternal or not. The entire universe is divided into three sets; the unit set that has sound as its only member, the set of eternal things and that of non-eternal things. Something is absent from both eternal and non-eternal entities other than sound, if it is absent from at least one of them. (ii) And the property should belong to the non-eternal entities (*anitya-vṛtti-dharma*) also. The property $P$ can be represented as the following:

$$(\text{absent from the eternals other than sound } \lor \text{ absent from the non-eternals other than sound}) \cdot \text{present in non-eternals}$$

A base case for the *mahāvidyā* is a pot.\(^{12}\) That means a pot should have both the Reason, i.e., knowability, and the Target, i.e., the complicated property $P$ we have discussed in the preceding paragraph. In fact, the main Co-sites, i.e., the non-eternal entities are Co-sites of the *mahāvidyā* too. So all non-eternal entities should have both the properties. The Reason is universally present. The question is, whether the Target is present in a pot that has several properties such as pot-ness, non-eternity etc. Let us check pot-ness. It resides in non-eternal things since it resides in pots. Thus the second criterion is satisfied. Now we should see whether pot-ness satisfies the first criterion. It is absent from eternal entities. Thus it is not present in both eternal and non-eternal entities other than sound. Therefore a pot has a property $P$ that satisfies both the criteria and $P$ happens to be pot-ness itself. Not only a pot, but a cloth, a tree, or a cup—in fact any non-eternal entity for that case—would also satisfy both the criteria. So infinite

\(^{12}\) Consider an inference: ‘This hill has fire, since it has smoke, like a kitchen’. In this, the phrase ‘like a kitchen’ represents a *dṛṣṭānta*, i.e., a base case. Likewise, in the *mahāvidyā* inference, ‘the soul has a property $P$, since the soul is knowable, like a pot’, a pot represents a base case.
non-eternals are loci of both knowability and the complex property $P$. They are our base cases. We would assume that another thing $x$ that has knowability should have $P$ too, unless there is some strong counter-condition.

A soul too has knowability. So it must have the complex property $P$ too. We have observed thousands of cases and seen that anything smoky is fiery. Now when we see a cloud of smoke on a hill, we definitely infer fire there, without checking whether there is fire on that hill or not. Such is the force of inference. Let us also assume that a soul has the property $P$. The question is, which one? It has many properties such as soul-ness, eternity, soul-pot-either-ness ($\text{ghatātmānyonyatva}$, i.e., the property of being either a soul or a pot, a property, which resides in both a soul and a pot), soul-sound-either-ness ($\text{śabdātmānyonyatva}$) etc. Can soul-ness be a good candidate for the property under consideration? No. Although it is not present in both eternal and non-eternal objects other than sound (as it is absent from non-eternals), it does not satisfy the second criterion. I.e., it does not belong to the non-eternals as it belongs to souls only. Could soul-pot-either-ness be a good candidate? It is there in a non-eternal since it is there in a pot. It is also there in an eternal entity since a soul is eternal. So it is present in both eternals and non-eternals other than sound. Thus it does not satisfy the first criterion, which is not being present in both.

If we check any soul-$y$-either-ness when $y$ is non-eternal, we shall find that none of them would satisfy both the criteria. Then two candidates are left; soul-$x$-either-ness when $x$ is an eternal entity other than a soul, and soul-sound-either-ness. Soul-$x$-

<table>
<thead>
<tr>
<th>Property</th>
<th>Property-holder</th>
<th>Presence in non-eternals (other than sound)</th>
<th>Presence in eternals (other than sound)</th>
<th>(i) Non-presence in both, i.e., absence in either</th>
<th>(ii) Presence in non-eternals</th>
<th>(i) and (ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pot-ness</td>
<td>Pot</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Soul-ness</td>
<td>Soul</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Soul-non-eternal-either-ness</td>
<td>Soul and non- eternal</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Soul-$x$-either-ness when $x$ is eternal other than soul</td>
<td>Soul and another eternal other than soul</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Soul-sound-either-ness</td>
<td>Soul and sound</td>
<td>-</td>
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<td>+</td>
</tr>
</tbody>
</table>
either-ness does not reside in any non-eternal entity. Thus it does not satisfy the second criterion. By the law of residual (pārisēsyā-nyāya), soul-sound-either-ness must fulfill both the criteria. No doubt it satisfies the first since it is not present in both the eternals and non-eternals other than sound by virtue of its absence from the non-eternals other than sound. It must belong to a non-eternal entity since it has to fulfill both the criteria. It resides in both a soul and sound, and nowhere else. A soul is not non-eternal. Then, once again by the law of residual, sound has to be non-eternal. Table 2 will summarize the above discussion.

The crux of the argument is the following. We inferentially know that a soul has the property $P$. We also know that a soul has properties $P_1, \ldots, P_n$. What we do not know is, which property is identical to $P$? We have checked and found that none of the properties $P_1, \ldots, P_{n-1}$ can be identical to $P$. That implies, through the law of residual, that $P_n$ is identical to $P$. $P_n$ has nothing that opposes the identity. Therefore, we infer that $P = P_n$. In order to satisfy the conditions of this identity, either the soul under consideration itself or sound has to be non-eternal. There is no doubt that the soul is eternal. Therefore, sound must be non-eternal.

Let us get back to the construction of the mahāvidyā. We have seen that the Target, i.e., the complex property $P$, is not present in both the main Co-site and main Anti-site. It must also be located in a locus that has the main Target, i.e., non-eternity. This was the second criterion. Finally, $P$ has been established in something that does not have the main Target, i.e., non-eternity. The Site of our mahāvidyā was a soul, which is not non-eternal.

Schema 2

pakṣāpakṣavipakṣānyavargād ekaikam uddhṛtaṁ, bhinnāṁ sādhyavatas tadavad-dhrtyāvadhibhedinaḥ. (ayaṁ ghaṭāḥ etadghaṭāṅkūrāṅyānasya kartāṅkāḥ.) Verse 6, DM13

Translation: The main Site, main non-sites (apakṣas, i.e., the [Target-possessing] non-sites, which are nothing but the Co-sites) and main Anti-sites are picked up (uddhṛta) and each of them is shown to be different from $x$ that possesses the Target (sādhyavat), and that is different from another Target-possessor that has been picked up. Example: This pot is different from a created entity that is different from anything other than this very pot or a sprout, [since it is knowable].14

Explanation: The main inference is: A sprout is created [by a sentient being] (sakartṛka), since $γ$. The main Site, main Co-sites and main Anti-sites are a sprout, the created entities such as pots and clothes, and the uncreated entities such as the sky and souls respectively. The main Target is created-ness (i.e., the property of being created by a sentient being).

13 Telang (1920, p. 155).

14 In all these mahāvidyā inferences, the Reason is knowability. The author mentions that in the first schema.

15 A non-site is a complement of the Site. Thus it is either a Co-site or an Anti-site. ‘Anti-cite’ is mentioned separately. Thus it has to be the Co-site. I think, for metrical reasons, Kulārka does not write ‘sapakṣa’, since he writes verses that have to maintain a specific metre.
The mahāvidyā inference has ‘picked up’ a sprout, a pot and all other things that include the uncreated ones. Thus it incorporates the main Site, main Co-site and main Anti-site.\textsuperscript{15} Each of them will be shown as something different from a created entity (namely ‘this pot’). Finally the inference proves that this pot too is different from another created entity, which in turn is different from anything other than this pot or a sprout. The pervasion is: anything that has knowability is different from a created entity other than anything that is different from this pot or a sprout.

First we shall see how all the created entities (of course other than the pot referred by the phrase ‘this pot’) satisfy the pervasion. Let us call the referent of ‘this pot’ \( p \). Now consider a cup. It is different from a created entity that in turn is different from anything that is a non-\( p \) and non-sprout, since it—the cup—is different from \( p \). \( p \) (i.e., \textit{this pot}) is a created entity and is different from anything other than \( p \) or a sprout. What is different from anything other than \( p \) or a sprout. It is either \( p \) or a sprout.\textsuperscript{16} A table, or a chair, or a mug is something other than \( p \) or a sprout. Thus any created entity (other than \( p \)), i.e., a main Co-site satisfies the pervasion.

Any uncreated entity such as the sky too satisfies the pervasion, since it is different from \( p \) (i.e., this pot). So the pervasion applies to a main Anti-site. Similarly the main Site, i.e., a sprout also satisfies the pervasion since it is different from \( p \).

The Site of the mahāvidyā must satisfy the pervasion, since everything else (i.e, every created or uncreated entity) does. The Site is \( p \) (i.e., \textit{this pot}) itself. It must be different from a created entity \( y \) that is different from anything other than \( p \) or a sprout. \( y \) is either \( p \) or a sprout. But this pot cannot be different from itself. Therefore it is different from a sprout. Thus \( y \) turns out to be a sprout. Remember that \( y \) is a created entity. That means a sprout is a created entity. It must have a creator and that creator is God. For no other sentient being can create a sprout.

Now we shall get back to the construction of this inference. The main Site, main Co-site and main Anti-site have been incorporated in the mahāvidyā and each of them has been shown to be different from an entity that possesses the main Target (i.e., created-ness). In these cases, that entity is ‘this pot’. But this pot itself is different from a created entity \( C \), which is either this pot or a sprout. \( C \) cannot be this pot since nothing is different from itself. Then \( C \) has to be a sprout. Since \( C \) is a created entity, a sprout too is created (by a sentient being). If I deny this, I deny the force of inference, since, as we have seen, every other entity (created or uncreated) satisfies the pervasion, ‘Anything that is knowable is different from \( x \), which is a created entity and which is different from anything other than this pot or a sprout.’ In all other cases, \( x \) is this pot itself. In the case of this pot, \( x \) happens to be a sprout.

\textsuperscript{16} Something different from anything other than \( z \), is \( z \) itself. By the same token, something different from anything other than this pot or a sprout is either this pot or a sprout.
Schema 3

In the first schema, a main Anti-site (a soul) became the Site of the associated mahāvidyā inference. In the second schema, a main Co-site (a pot) became the Site of the associated mahāvidyā inference. In the third schema, the Site of the mahāvidyā will be the main Site.

\[ \text{pakṣāpakṣagatād anyat sādhyavādṛtī pakṣagam. (śabdāḥ śabdāśabdāvṛtityan-}\]
\[ \text{ityavṛttidharmavān.) Verse 4, DM}^{17}\]

Translation: Both the main Site and its complement (apakṣa) will not have the Target that is there in a thing that possesses the main Target. Such a Target is going to be established in the main Site. Example: A sound-individual has a property (i) which is not present in both sounds and non-sounds, and (ii) which is there in a non-eternal entity [since it is knowable].

Explanation: The main inference is once again, ‘A sound-individual is non-eternal, since γ’. We begin with the example. The pervasion is: Anything that is knowable has a property (i) which is not present in both sounds and non-sounds, and (ii) which is there in a non-eternal entity. First we try the main Co-sites, i.e., the non-etransals such as a pot. A pot has pot-ness, which is not present in both sounds and non-sounds, since pot-ness is absent from sounds. Thus the first criterion is met. A pot itself is non-eternal. So pot-ness meets the second criterion by being present in the pots. Thus all non-eternal entities satisfy the pervasion.

We then take up eternals such as a soul. This case is a bit tricky. A soul has soul-ness, which is not there in non-eternal things. Thus soul-ness does not satisfy the pervasion, since it does not satisfy the second criterion. A soul has soul-sound-either-ness also. It does not fulfill the first criterion since it is present in both sounds and souls, which are non-sounds. But a soul has another property, (soul)-(non-sound-non-eternal)-either-ness, whose special case is soul-pot-either-ness.\(^{18}\) It is not present in both sounds and non-sounds, since it is absent from sounds. It is present in a non-eternal entity since it is present in a pot. Thus it fulfills both the criteria.

We have seen that both the eternal and non-eternal entities satisfy the pervasion. So it must also apply to a sound-individual. A sound-individual has (sound)-(non-sound)-either-ness, whose special case is sound-pot-either-ness. But the properties of this group do not satisfy the first criterion. They oppose the pervasion. That means we have to look for other properties. Except sound-ness no other property satisfies the first criterion, since other properties such as entity-ness or attribute-ness \((guṇatva)\) belong to non-sounds too. Thus sound-ness, or for that case any property that resides in sounds only, is the only possible candidate. For all other properties clearly oppose the pervasion that has been already established. That being the case, sound-ness must also reside in a non-eternal entity. Since the non-eternal entity cannot be any non-sound, it is a sound-individual. Thus sound is non-eternal.

\(^{17}\) Telang (1920, p. 155).

\(^{18}\) A pot is a non-eternal non-sound.
General Structure and Features

Disjunction (or, which is represented by ‘∨’) has been used very innovatively in the mahāvidyā inferences. The Target here is like a variable. It is an abstract property that relates itself to the main Site, main Co-site and main Anti-site in different ways. I.e., it is just a description that fits different properties in different cases. It can be compared to ‘the property of being either Brown or Jones or Robinson’.\(^{19}\) It can be used to give a definition by intension of a class of other specific properties.

We may consider the third schema that can be re-written as the following:

\[
\sim \text{ (residing in sounds} \cdot \text{ residing in non-sounds)} \cdot \text{residing in non-eternals } [a] \\
\equiv (\sim \text{ residing in sounds } \lor \sim \text{ residing in non-sounds}) \cdot \text{residing in non-eternals } [b]
\]

[b] is basically a conjunction whose first conjunct is a disjunction (i.e., (\sim \text{ residing in sounds } \lor \sim \text{ residing in non-sounds})). In order to satisfy a conjunction \((p \cdot q)\), an entity must satisfy both the conjuncts \(p\) and \(q\). In order to satisfy a disjunction \((p \lor q)\), it is enough if an entity satisfies at least one of the disjuncts.

(1) In the case of the non-eternal entity \(x\), \(x\)-(ness) satisfies [b] in the following way. Let us consider a non-eternal entity, say a pot. Pot-ness satisfies the second conjunct (= residing in non-eternals) by residing in a non-eternal entity, namely a pot. It satisfies the first conjunct (= (\sim residing in sounds \lor \sim residing in non-sounds)) by residing in a non-sound, namely a pot once again. Basically it satisfies the second disjunct (= \sim residing in non-sounds), and by doing that it satisfies the entire disjunction.

(2) In the case of the eternal entity \(y\), \(y\)-(non-sound)-either-ness satisfies [b] in the following way. Let us consider an eternal entity, say time. Time-(non-sound)-either-ness resides in time and non-sounds only. It satisfies the second conjunct (= residing in non-eternals) by residing in a non-eternal entity such as a pot, which is a non-eternal non-sound. It satisfies the first conjunct (= (\sim residing in sounds \lor \sim residing in non-sounds)) by not residing in sound. It satisfies the first disjunct (= \sim residing in sounds), and by doing that it satisfies the entire disjunction.

(3) Since all the entities, eternal and non-eternal, satisfy [b], we assume that a sound-individual \(s\) too would do the same. It is well-known that \(s\) satisfies the first conjunct by virtue of having sound-ness that does not reside in non-sounds. Then it must also satisfy the second conjunct. Therefore it has to be non-eternal.

\(^{19}\) Russell (1920, p. 12):
Brown, Jones, and Robinson all of them possess a certain property which is possessed by nothing else in the whole universe, namely, the property of being either Brown or Jones or Robinson. This property can be used to give a definition by intension of the class consisting of Brown and Jones and Robinson.

\(^{20}\) This is roughly the gist of the following: pakṣe prakāraṇatāreṇa sādhvāpasamhārasālīte sati deśānte prakāraṇatāreṇa sādhvāpasamhārasālītvam mahāvidyānumānakvam. Quoted in Bhattacharya and Bhattacharya (1981, p. 104), The entry on Mahāvidyānumāna.
Thus, a mahāvidyā inference is that whose Target gets related to the Site and base cases in different ways. We can say that the first essential feature of a mahāvidyā inference is:

(I) The Target turns out to be different properties in different loci in different ways.

Although disjunction plays an important role in the formation of most of the mahāvidyā inferences, it is not essential to them.

Secondly, a mahāvidyā inference does not have any Anti-site, since the Reason is present everywhere. In all the cases we discussed, the Reason is knowability that belongs to every entity (by assumption). But the associated main inference always has both Co-sites and Anti-sites. Therefore the second essential feature of a mahāvidyā inference is:

(II) Its Target is shown to be present in all entities.

The demolition of Anti-sites is due to the force of inference. Somehow I have been able to show that the Target is present in all other cases. So I assume that it must be there in the Site. I just look for that property of the Site which does not oppose the Target-conditions.

The Antidote to Mahāvidyā Inferences

When does one say that an inference is faulty? When it has got an inferential defect (hetvābhāsa). An inferential defect is roughly a cognitive content (viṣaya) (i) that contradicts an inference, and (ii) that belongs to a piece of cognition whose invalidity has not been established (agṛhīta-aprāmāṇyaka). A piece of cognition whose invalidity has not been established may be called an un-invalidated cognition. Such a cognition is either a valid one, or something, which is not invalidated so far.

Suppose somebody infers, ‘This ostrich must be able to fly, since it is a bird.’ The pervasion-claim of this inference is: anything that has bird-ness has the ability to fly. The content of the following valid cognition contradicts the pervasion-claim: ‘It is not true that anything that has bird-ness has the ability to fly. An emu is a flightless bird’. Thus the content of the contradictory cognition, i.e., the very fact that an emu cannot fly despite being a bird, is the defect of the inference. A defect of a pervasion-claim is called ‘deviation’ (vyabhicāra).

Suppose somebody infers, ‘this hill has fire since it has smoke’. The inference demands that its Site has the Reason. But I check every square inch of the hill and do not find smoke there. As the Reason is absent from the Site, the Site-located-ness-claim (or paksadharmatā-claim) of the inference is contradicted. The content of the valid cognition, ‘there is no smoke on the hill’, which correspondence to the

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21 For example, we may consider the seventh mahāvidyā in DM (Telang 1920, p. 155): [A] locus of [a] sound-individual is different from [another] locus (śabdādhirakaraṇam śabdādhirakarṇād anyat). I am not going to explain it.
factual absence of smoke on that hill, becomes the defect of the inference. Such a defect is a kind of asiddhi (non-establishment).

The rivaling cognition need not always be a valid one. Let us consider the following example. A physician has observed that anybody, who eats strawberries everyday, suffers from a disease called Coccinistercus. She has also observed that anybody, who eats blackberries everyday, never suffers from that disease. She has not found any counterexample to either of those observations. Now she comes across a small boy, who eats both strawberries and blackberries every day. There is absolutely no way she can check the boy for the symptoms of Coccinistercus. What will she conclude? Two different practices of the boy insist her to form two contradictory conclusions. One must be wrong. But she does not know which one is wrong. Both the contradictory Reasons are equally strong and they counterbalance each other. In this case, both the inferences are faulty. This defect is called ‘counterbalancing’ (satpratipakṣa).

We may note here that there is difference between reductio ad absurdum (whose Indian counterpart is prasaṅga tarka) and counterbalancing. In a reductio proof, one assumes X, then shows that the assumption leads one to a formal contradiction; thus ~X is proven. It is a formal technique. But counterbalancing is an epistemic idea. In this case, two inferences ‘x is y, since z’ and ‘x is not y, since w’ seem to work fine separately; but when both apply to a single object, they counterbalance each other.

There is another inferential defect called upādhi. Suppose, after considering numbers like 20, 30 etc., somebody hurriedly hypothesizes that:

[1] Any multiple of 5 is a multiple of 10.

They also know that:

[2] Any multiple of 10 is an even number.

[1] and [2] imply:

[3] Any multiple of 5 is an even number.

But they come across odd numbers like 15 or 25, which contradict [3], and finally realize:

[4] It is not true that any multiple of 5 is an even number.

[4] validly claims that despite pervading the property of being a multiple of 10, even-ness does not pervade the property of being a multiple of 5, since 25 is a multiple of 5 without being an even number. In this case, with respect to [1], even-ness is an upādhi or ‘Inferential undercutting condition’. The discovery of an upādhi invalidates an inference.

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22 Any case of being a multiple of 5 is a case of being a multiple of 10.
23 Y pervades X, if and only if all cases of X are cases of Y.
24 This translation is due to Phillips and Tatacharya (2002).
Pervasion is transitive. I.e., if all the cases of \( x \) are cases of \( y \), and all the cases of \( y \) are cases of \( z \), then all the cases of \( x \) are cases of \( z \) too. An \( \text{upādhi} \) goes against the transitivity of pervasion. Thus it is an anti-transitive factor.

In the following subsection, we shall see how the opponents of \( \text{mahāvidyā} \) inferences find those defective. \( \text{Mahāvidyāviḍambanam} \) by Mahādeva is a long text with three long chapters. The text discusses several \( \text{mahāvidyā} \) inferences and detects their defects. Bhuvanasundara summarizes it in his \( \text{LMV} \). Our discussion will follow \( \text{LMV} \).

**Laghumahāvidyāviḍambana (LMV) by Bhuvanasundara**

It is evident that when the opponent detects and points out the defects of a \( \text{mahāvidyā} \) inference, they defeat the inference. Both Mahādeva and Bhuvanasundara think that there is no property that belongs to everything. But the Reason of a \( \text{mahāvidyā} \) is always a universally located \( (\text{kevalānvayi}) \) property. Apart from that, there are the following defects.

1. **Over-generalization (\( \text{atiprasaṅgaduṣana} \)):** Suppose a \( \text{mahāvidyā} \) inference has attacked your position. If you modify the inference a bit, it will establish something your opponent would consider undesired, and support you. For example, let us remember the second schema: This pot is different from a created entity that is different from anything other than this very pot or a sprout. It proves that a sprout has a sentient creator. Since no ordinary sentient being can create a sprout, the creator must be God Himself. But the Naiyāyika, who uses this \( \text{mahāvidyā} \) against you, believes that no other creator has created God. You just replace ‘a sprout’ by ‘God’.\(^{25}\) Now it proves that God is a created entity. Thus a \( \text{mahāvidyā} \) can be turned against the person, who uses it.

2. **Inferential Undercutting Condition (\( \text{upādhi} \)):** According to Bhuvanasundara, the \( \text{upādhi} \) of the inference, ‘the soul has a property \( P \) such that (i) \( P \) does not reside in both eternal and non-eternal entities other than sound, and (ii) \( P \) resides in a non-eternal entity, since the soul is knowable, like a pot’ is inaudibility \( (\text{aśrāvanatva}) \). Only a sound-individual is audible. And here the non-eternity of sound is going to be established. It has not yet been established. So one can validly say that anything that is non-eternal is in-audible.\(^{26}\) In the \( \text{mahāvidyā} \) inference, the underlying pervasion is: anything that is knowable has the property \( P \). The Target and Reason are \( P \) and knowability respectively. The property \( P \) is nothing but non-eternity because finally it establishes non-eternity. If knowability is pervaded by non-eternity, and non-eternity is pervaded by in-audibility, then knowability must be pervaded by in-audibility. But it is not true that anything that is knowable is in-audible too, because a

\(^{25}\) \( \text{ayaṃ ghaṭaḥ etadghaṭeśvarānyasyasakarṭkānyah meyatvāt ghaṭavat.} \ \text{LMV}, \ \text{Telang} \ (1920, \ \text{p. 153}). \)

\(^{26}\) \( \text{yad yat anityam tat tad aśrāvanam.} \ \text{LMV}, \ \text{Ibid.}, \ \text{p. 152}). \)

\(^{27}\) \( \text{atha atrāṇityatvam sādiḥyam na bhavati, kiṁtu śabdetarāṇityaniṣṭetvādikam. tasya cpādhinā vyāptir nāsti. śabdatvena vyabhicārād iti nāyam upādhir iti cet. maivam. asya hanumalloalāṅgalāmbasyāpi anitye eva visrāmāt.} \ \text{Ibid.}. \)

\( \text{Springer} \)
sound-individual is knowable without being in-audible. Thus in-audibility is the upādhi for the mahāvidyā.

Note: Bhuvanasundara writes: ‘[My opponent may say that,] ‘Well, here the Target is not non-eternity, but it is the special property \( P \), i.e., a property that does not reside in both eternal and non-eternal entities other than sound etc. And \( P \) is not pervaded by the alleged upādhi, [i.e., in-audibility]. For a sound-individual is a counterexample to such a pervasion. Hence there is no upādhi’. I shall answer, no. The property \( P \), which is as long as the great tail of Hanumān, ends up being non-eternity. The Target is that, which finally gets established in the Site. And here it is nothing but non-eternity’. 27

I do not agree with him. As I mentioned before, the Target is like a variable here. It is a definition by intension that defines a set consisting of two properties. In the case of a non-eternal entity, say \( x \), it is \( x \)-ness. In the case of a soul, it is soul-sound-either-ness. Thus the charge of upādhi is perhaps not right.

(3) Vicious Circle (ātmāśraya) or Counterexample (vyabhicāra): ‘Does the property \( P \) belong to the Target of the mahāvidyā inference, i.e., \( P \) itself? If it does, then there would be a vicious circle. If it does not, then there would be a counterexample to the pervasion, ‘anything that is knowable has \( P \)’, since \( P \) would have both knowability and the absence of \( P \).’ 28

(4) Counterbalancing (satpratipakṣa): I think this is the most important point against the mahāvidyā inference. The other objections help us understand that something is wrong with the inference. But this one points out the formal defect mahāvidyā has in terms of the epistemology of anumāna. After having known the co-presence of \( x \) and \( y \) in various things, I have concluded that every locus of \( x \) is a locus of \( y \). When I see \( x \) somewhere, I may validly infer \( y \) if only my inference is free of inferential defects (hetvābhāsas). But a mahāvidyā inference suffers from counterbalancing.

By using the mahāvidyā techniques, one infers that a sound-individual is not-eternal since it has a property \( P \) (i) which is not present in both sounds and non-sounds, and (ii) which is there in an non-eternal entity, since it is knowable. 29 We ignore the Reason of the mahāvidyā part (i.e., since it is knowable) since it is an omnipresent property. We can form the counterbalancing inference by following a couple of steps:

[1] Replace all the occurrences of ‘eternal’ by ‘non-eternal’ and vice versa.

Thus we get:

A sound-individual is eternal since it has a property \( P \) (i) which is not present in both sounds and non-sounds, and (ii) which is there in an eternal entity.

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28 mahāvidyāsādhye mahāvidyāsādhyam vidyate na vā, vidyate cet, ātmāśrayah, no cet tarhi mahāvidyāsādhye meyvatvahetor vartanāt mahāvidyāsādhyasya ca avartanāt, tasyaiva vipakṣatvena anaitkāntika eva ayaṃ hetuḥ. Ibid.

29 See Schema 3.

Thus we get:

A sound-individual is eternal, since it has a property $Q$ (i) which is not present in both sounds and non-sounds, and (ii) which is there in an eternal entity.

We shall test whether the modified form really counterbalances the original inference.

1. In the case of an eternal entity, say a soul, soul-ness is $Q$.
2. In the case of a non-eternal entity, say a pot, pot-(non-sound)-either-ness is $Q$ [since at least one non-sound must be eternal, e.g., time].
3. In the case of a sound-individual, $Q$ is sound-ness itself.
4. Therefore, a sound-individual is eternal.\(^{30}\)

One, who wants to prove the non-eternity of sound by using another mahāvidyā, infers that sound is non-eternal, because a soul has a property $P$ such that (i) $P$ does not reside in both eternal and non-eternal entities other than sound, and (ii) $P$ resides in a non-eternal entity. The Site is a locus that has both the Reason and Target. In the above inference the Reason (i.e., a soul has a property $P$ such that (i) $P$ does not reside in both eternal and non-eternal entities other than sound, and (ii) $P$ resides in a non-eternal entity) does not seem to belong to the Site, i.e., ‘sound’. If one wants to formally see ‘sound’ as the Site of the inference, one may consider the following:

Sound is non-eternal, because it is identical to $x$ such that a soul has a property $P$ such that (i) $P$ does not reside in both eternal and non-eternal entities other than $x$, and (ii) $P$ resides in a non-eternal entity.

Its counterbalancing inference is:

Sound is eternal, because it is identical to $x$ such that a pot has a property $Q$ such that (i) $Q$ does not reside in both eternal and non-eternal entities other than $x$, and (ii) $Q$ resides in an eternal entity.\(^{31}\)

I leave it to the reader to write down the steps for forming the above counterbalancing inference. I have shown two counterbalanced cases. But it is easy to see that all the mahāvidyā inferences are counterbalanced.

**Mahāvidyā vs. Diagonal Arguments**

I shall try to show here that diagonal arguments share some essential features of the mahāvidyā inference. I shall choose Cantor’s diagonal argument (1892), presented in his German paper entitled *Ueber eine elementare Frage der Mannigfaltigkeitslehre*, as

\(^{30}\) See ‘General Structure and Features’ in order to understand this fully.

\(^{31}\) *ghataḥ śabdetarāṇityavṛttiivaṇadhikaraṇaṇityavṛttiidharmavān. LMV*, Telang (1920, p. 152).

The property $Q$ is there in a soul because it has soul-ness, which does not reside in both eternals and non-eternals other than sound and which belongs to souls that are eternal. When $Q$ gets related to a pot, it becomes pot-sound-either-ness.

a representative of diagonal arguments. Disproving the argument is not my concern. I would try to explain why many of us feel uncomfortable with the proof.

The Argument

Let me write down the proof first. My re-presentation is based on Lipton and Regan (2013, pp. 84–85). Consider the following array of infinite rows:

\[
\begin{align*}
&\begin{array}{ccccccc}
  & s_1^1 & s_1^2 & s_1^3 & s_1^4 & \cdots & s_1^n \\
 1 & s_2^1 & s_2^2 & s_2^3 & s_2^4 & \cdots & s_2^n \\
 2 & s_3^1 & s_3^2 & s_3^3 & s_3^4 & \cdots & s_3^n \\
 3 & s_4^1 & s_4^2 & s_4^3 & s_4^4 & \cdots & s_4^n \\
\end{array}
\end{align*}
\]

Thus the \(i\)th row is

\[
\begin{align*}
&\begin{array}{ccccccc}
  & s_i^1 & s_i^2 & s_i^3 & s_i^4 & \cdots & s_i^n \\
 1 & s_{i+1}^1 & s_{i+1}^2 & s_{i+1}^3 & s_{i+1}^4 & \cdots & s_{i+1}^n \\
 2 & s_{i+2}^1 & s_{i+2}^2 & s_{i+2}^3 & s_{i+2}^4 & \cdots & s_{i+2}^n \\
 3 & s_{i+3}^1 & s_{i+3}^2 & s_{i+3}^3 & s_{i+3}^4 & \cdots & s_{i+3}^n \\
\end{array}
\end{align*}
\]

where each \(s_i^j\) is either 1 or 0.

Suppose \(S\) is the set of all sequences of 0 and 1. Our plan is to construct an infinite sequence \(t\) that consists of 0 and 1 only. But we need that \(t\) is different from any row of the array given above, i.e., different from every \(s\). Let \(t\) be a sequence such that:

\[
t(m) = \neg s^n(m) \text{ where } \neg 0 = 1 \text{ and } \neg 1 = 0.
\]

Thus \(t \neq s^1\) since \(t(1) = \neg s^1(1)\); \(t \neq s^2\) since \(t(2) = \neg s^2(2)\) and so on. It is evident that ‘\(t\) is just equal to the negation of the diagonal elements’:

\[
\begin{align*}
&\begin{array}{ccccccc}
  & s_1^1 & s_1^2 & s_1^3 & s_1^4 & \cdots & s_1^n \\
 1 & s_2^1 & s_2^2 & s_2^3 & s_2^4 & \cdots & s_2^n \\
 2 & s_3^1 & s_3^2 & s_3^3 & s_3^4 & \cdots & s_3^n \\
 3 & s_4^1 & s_4^2 & s_4^3 & s_4^4 & \cdots & s_4^n \\
\end{array}
\end{align*}
\]

Thus we have proven that despite being a sequence of 1 and 0, \(t\) is not part of the array, i.e., it is not identical to any \(s\). The consequence of this proof is very significant: \(S\) is uncountable. I.e., there is no one-to-one mapping between the set of natural numbers and \(S\). Thus ‘they have different cardinalities.’

Refutations and Defense

Many contemporaries of Cantor felt uncomfortable with his proof. Among them, two important names are Leopold Kronecker and Henri Poincaré. Poincaré thought that the main defect of the diagonal argument was due to its non-predicative nature.

33 Lipton and Regan (2013, p. 84).
He (2009, p. 190) writes: ‘The definitions that must be regarded as non-predicative are those which contain a vicious circle.’ He (2009, p. 194) further writes:

In these definitions we find the word *all*, as we saw in the examples quoted above. The word *all* has a very precise meaning when it is a question of a finite number of objects; but for it still to have a precise meaning when the number of the objects is infinite, it is necessary that there should exist an actual infinity. Otherwise *all* these objects cannot be conceived as existing prior to their definition, and then, if the definition of a notion N depends on *all* the objects A, it may be tainted with the vicious circle, if among the objects A there is one that cannot be defined without bringing in the notion N itself.

According to Hodges (1998, p. 6), Wittgenstein claimed that Cantor’s argument has no deductive content at all; he (Wittgenstein) singles out Cantor’s argument because it would appear to have no relation to any imaginable activity. Wittgenstein (1956, p. 129) writes:

Surely – if anyone tried day-in day-out ‘to pull all irrational numbers into a series’ we would say: “Leave it alone; it means nothing; don’t you see, if you established a series, I would come along with the diagonal series!” This might get him to abandon his undertaking. Well, that would be useful. And it strikes me as if this was the whole and proper purpose of the paper.

I shall discuss the comments of Poincaré and Wittgenstein later. The point I would like to make here is: the disquiet over Cantor’s proof does not seem to have disappeared. Hodges (1998) discusses and summarizes a number of unpublished papers that attempted to refute the proof. According to him, none of those was successful. But he (1998, p. 3) too feels that something seems to be wrong with Cantor’s proof:

This argument is often the first mathematical argument that people meet in which the conclusion bears no relation to anything in their practical experience or their visual imagination. Compare it with two other simple facts of cardinal arithmetic. First, \( m \times n = n \times m \). We can see what this amounts to by thinking of a rectangle with one side of length \( m \) and one side of length \( n \). The picture points to the right formal argument when \( m \) and \( n \) are finite, and exactly the same argument works when they are infinite……

But then we come to Cantor’s result, and all intuition fails us. Until Cantor first proved his theorem…., nothing like its conclusion was in anybody’s mind’s eye. And now we accept it because it is proved, not for any other reason.

But ultimately Hodges defends Cantor. Poincaré seems to say (if I follow him correctly) that Cantor’s proof involves infinity; but ‘it is necessary that there should exist an actual infinity’. The charge against infinity is rightly addressed in Hodges (1998, p. 12):

One author complained that Cantor’s proof requires us to write out an infinite diagram. But that’s a thing we can’t do; the author conscientiously proves this
as follows. As we make the list, it becomes infinite either gradually, or
suddenly, or not at all. The idea that it becomes infinite gradually is
incoherent; at any stage it is either definitely finite or definitely infinite. If it
suddenly becomes infinite, there is a stage at which it becomes infinite. But
this is false; at every stage in the construction of the list, it is finite. Therefore
it never becomes infinite.
Of course nobody would suggest that in order to carry out Cantor’s proof you
actually have to write out the infinite diagram, would they? Would they?
I too think that infinity is not a problem here; the problem lies somewhere else. It is
probably related to the structure of Cantor’s proof. According to Hodges, a
deductive argument should have a few essential features. Here is an extremely
illuminating passage from Hodges (1998, p. 6):
Broadly speaking, such an argument has three kinds of component:

- There are the stated conclusions, the stated or implied starting assumptions, and
  the intermediate propositions used in getting from the assumptions to the
  conclusion. I shall call these the object sentences.
- There are stated or implied justifications for putting the object sentences in the
  places where they appear. For example if the argument says ‘A, therefore B’, the
  arguer is claiming that B follows from A.
- There are instructions to do certain things which are needed for the proof. Thus
  ‘Suppose C’, ‘Draw the following picture, and consider the circles D and E’,
  ‘Define F as follows’.

If one wants to find fault with an argument a, one has to show that at least one of
these aspects is problematic in a. Hodges (1998, p. 6) reports that none of the attacks
on Cantor’s proof targeted its object sentences; but some of them had issues against
its justifications and instructions. But he thinks that none of those attacks could
actually refute the proof and concludes that ‘there is nothing wrong with Cantor’s
argument’.35

Cantor’s Proof from an Indian Epistemic Perspective

When I try to read Cantor’s proof, I find some of its object sentences and
constructions problematic. Probably it is due to my conviction that the epistemo-
logical arguments against the mahāvidyā inference are correct.

Hodges (1998, p. 4) writes, ‘It was surprising how many of our authors failed to
realize that to attack an argument, you must find something wrong in it.’ I shall try to
show what exactly seem to me problematic in Cantor’s proof.

1. By definition, the following is true:

   \[ t(m) = \neg s^m(m) \]
   This is an instruction.

2. The sequence \( t \) consists of 0 and 1. Thus it is a member of \( S \) (i.e., the set of sequences of 0 and 1). That being the case, \( t \) must have a serial number since it has a unique position in \( S \). Suppose the number is \( p \). By definition, each member of \( S \) is an \( s \). In that case,

\[ [b] \quad t = s^p \]

[b] is an object sentence since it is implied by the construction of \( t \).

3. Suppose \( s^p(p) = 0 \). From [b], it follows that

\[ [c] \quad t(p) = 0, \text{ since } t = s^p. \]

4. By assumption, \( s^p(p) = 0 \). Then,

\[ [d] \quad t(p) = \neg s^p(p) = 1, \text{ since } [a] \ t(m) = \neg s^m(m), \text{ when } m \text{ is any integer.} \]

5. We can see that [c] and [d] counterbalance each other, and as a consequence neither can be asserted.

6. The same argument works when \( s^p(p) = 1 \).

If we are not able to infer what is there at the \( p \)th position of \( t \), \( t \) would look like:

\[ \ldots \]

No inference guarantees that the \( p \)th digit of \( t \) must be either 0 or 1, since no valid inference is possible here. Hence we cannot validly say that the \( p \)th digit of \( t \) must be either 0 or 1. We can at most say that it is a sequence of 1 and 0 with a hole at its \( p \)th position. If this is true, then we cannot prove the theorem, ‘\( S \), i.e., the set of the sequences of 1 and 0 is uncountable’, which is the main inference (\( mūlānumāna \)) in this case. Of course we can prove that if \( t \) is constructed in the stipulated way, then it is not a member of \( S \), and if it is a member of \( S \), then it is not constructed in the stipulated way.

Cantor’s supporters may defend him by saying the following: ‘We are not saying that \( t \) is identical to an \( s \) (i.e., a member of \( S \)). In fact, we claim that it is not. We construct \( t \) just by negating the diagonal elements (i.e., \( s^1(1) \ s^2(2) \ s^3(3) \ s^4(4) \ldots \)).’ My response to them would be this: The moment you construct \( t \) as a sequence of 1 and 0, it obtains a serial number in \( S \), just by virtue of being a sequence of 1 and 0. Thus it ‘becomes’ an \( s^{\text{something}} \). This, according to Poincaré, is the non-predicative-ness of Cantor’s proof. The proof seems to work because it hides (or ignores) this point. We have an intuition that a member of a set \( \Sigma \) occupies a unique position in \( \Sigma \); hence it has a serial number in \( \Sigma \). The supporters of Cantor may refuse to accept this important piece of intuition. But an epistemologist may not do so. This intuition causes the disquiet over Cantor’s proof.

I re-quote Wittgenstein (1956, p. 129): “Leave it alone; it means nothing; don’t you see, if you established a series, I would come along with the diagonal series!” And the moment I come along with the diagonal series, it cannot help being part of the main series itself.
Reading Cantor from the Anti-mahāvidyā Perspective and Vice versa

I begin this subsection with a (relevant) personal experience. The other day I was talking to one of my relatives through Skype. He is a brilliant mathematician. At some heated moment, I said, ‘Why should I every time do what you say?’ He said, with an angry tone, ‘Don’t do what I say, please’. I asked him, ‘Do you realize that you have said something stupid?’ He took a couple of moments and responded: ‘My God! OK, except this, ignore everything else I say’. What I learned from this event is the following. When we say something self-referential, we normally tend to ignore the fact that what we are saying falls under its own scope of denial or assertion. I think that Cantor’s proof exhibits the same tendency. Epimenides the Cretan too did the same thing.

How does one infer the digit at the \(j\)th position of \(t\)? Definitely by using \([a]\) (i.e., \(t(m) = \neg s^m(m)\) when \(m\) is any integer). The method works fine till the \((p - 1)\)th position of \(t\). [We may remember that \(p\) is \(t\)’s own serial number. If we deny that \(t\) has a serial number in \(S\), we intuitively deny that \(t\) is a member of \(S\).] So we assume that it would work fine at every position of \(t\), including the \(p\)th position. But contrary to our assumption, the method fails when it reaches the \(p\)th digit, since counterbalancing invalidates the inferential method. Similarly a mahāvidyā inference constructs a property—a definition by intension—that picks up different specific properties (dharmas) in different cases. It fits a property \(d_1\) that belongs to the main Co-sites. It fits \(d_2\) that belongs to the main Anti-sites. So we assume that it must fit some property in every case, since the main Co-sites and main Anti-sites normally include all entities except the Site of the mahāvidyā.\(^{36}\) Hence, we assume, it must fit the dharma \(d_s\) that would belong to the Site. This assumption, we have seen, is unwarranted, and an inference counterbalances it.

The inference that establishes the uncountability of \(S\) is the following: \(t\) belongs to the set \(S\) and is different from any \(s\) (i.e., any sequence of 1 and 0), since \(t\) happens to be a sequence of 1 and 0, and, by construction, \(t\)’s \(m\)th digit is complementary to the \(m\)th digit of \(s^m\). This inference seems to be valid, because the proof never mentions—even forgets—\(p\), the unfortunate serial number of \(t\). The mahāvidyā inference too forgets that it provides its enemy with a counterbalancing inference.

The Contribution Made by Cantor’s Proof and Mahāvidyā

Apart from fooling their opponents, what does the Naiyāyika achieve by inventing the mahāvidyā inference? In my humble opinion, they do achieve something else also. They present us with a smart argument pregnant with its own killer. In fact the argument hides its killer inside its own womb. That is why it was so difficult for others to refute a mahāvidyā inference. An ordinary counterbalanced inference does not formally carry its own rival. There is nothing in the form of the inference, ‘This

\(^{36}\) That means that the Target of the mahāvidyā inference will be present in every entity. This is equivalent to saying that the inference has no Anti-site. We must not confuse the mahāvidyā Anti-site and the main Anti-site that belongs to the main inference.
boy has *Coccinistercus* since he eats strawberries everyday’, that presents its equally potent rival, i.e., ‘This boy does not have *Coccinistercus* since he eats blackberries everyday’. On the contrary, a little modification of the form of the *mahāvidyā* inference μ gives the counterbalancing argument. It is almost a derivation.

The diagonal argument too shares this self-betraying tendency by carrying its own invalidator. A contradiction is hidden in the construction of t itself. Cantor’s diagonal argument proves something very convincingly by hiding an implication of one of its components.

**Conclusion**

For a few centuries, many Indian debaters used the *mahāvidyā* inference against their opponents.37 The opponents felt helpless because they did not have an epistemological tool to fight the inference. Some authors provided them with a weapon against *mahāvidyā*. That weapon was the discovery of the inferential defects of a *mahāvidyā* inference itself. Among those authors, Mahādeva was perhaps the most competent. That is why nobody used *mahāvidyā* inferences after the thirteenth century.

On the contrary, the refutations of Cantor’s diagonal argument have not been well accepted in the western logical tradition. Wittgenstein expressed his discontent with Cantor’s argument. But his own diagonal argument seems to be a variant of Cantor’s.38

Maybe the mathematician will find no defect in Cantor’s proof. It is probably because they are happy to play their game without violating its preset rules. It is all about setting the rules and playing the game accordingly. Intuitive or epistemic disquiet is not their concern. Hodges (1998, p. 3) makes an interesting comment:

> There is a point of culture here. Several of our authors said they had trained as philosophers, and I suspect that most of them had. In English-speaking philosophy (and much European philosophy too) you are taught not to take anything on trust, particularly if it seems obvious and undeniable…. Mathematics is not like that; one has to accept some facts as given and not up for argument.

I think he is right. But if a philosopher—especially an epistemologist—can come up with a framework that may pin down an epistemological defect in Cantor’s proof, the framework would account for the intuitive disquiet over the proof. The anti-*mahāvidyā* arguments provide us with such a framework. It does not claim that uncountability cannot be a real property, or that the transfinite numbers do not exist, or that no actual infinity is possible. It merely says that we cannot validly claim t to be a sequence of 1 and 0, if we construct t in the stipulated way. I.e., our epistemic mechanisms do not allow us to validly claim that a sequence of 1 and 0 can obey the

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37 For various applications of *mahāvidyā* inferences in other texts, see Telang (1920, pp. 35-8) and Bhattacharya and Bhattacharya (1981, p. 103–104).

38 Floyd (2012).
construction rule stated by Cantor. This framework may be useful for somebody, who does not feel happy in the paradise created by Cantor.

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Abbreviations

DM  Daśaślokīmahāvidyāsūtra by Kulārka Paṇḍita (eleventh century)
MV  Mahāvidyāvivaraṇam, a commentary on DM by an unknown author
MVT  Mahāvidyāvivaraṇaṭīppanam, a commentary on MV by Bhuvanasundarāsūri (fifteenth century)
MVD  Mahāvidyāvidambanam by Mahādeva Bhaṭṭavādīndra (thirteenth century)
MVDV  Mahāvidyāvidambanavṛtti, a commentary on MVD by Bhuvanasundarāsūri
LMV  Laghumahāvidyāvidambanam, a summary of MVD by Bhuvanasundarāsūri

References