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First order Robust Controller Design for the Unstable Process with Dead Time

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Abstract
In this paper, the problem of stabilizing a given but arbitrary linear time invariant continuous time system with the transfer functions $P(s) = \frac{N(s)}{D(s)}$*, by a first order feedback controller* $C = \frac{x_1 s + x_2}{s + x_3}$ *was taken. The complete set of stabilizing controllers is determined in the controller parameter space* $[x_1, x_2, x_3]$*. This includes an answer to the existence question of whether P(s) is "first order stabilizable" or not. The set is shown to be computable explicitly, for fixed* x_3*.The results to stabilize lower order plants is extended to determine the subset of controllers which also satisfy various robustness and performance specifications. The problem is solved by converting the* H_∞ *problem into the simultaneous stabilization of the closed loop characteristic polynomial. The stability boundary of each of these polynomials can be computed explicitly for fixed* x_3 *by solving linear equations. The union of the resulting stability regions yields the set of all set of all* X_1 *and* X_2*.The entire three dimensional set is obtained by sweeping* X_3 *over the stabilizing range. They demonstrate that the shape of the stabilizing set in the controller parameter space is quite different and much more complicated compared to that of the PID controllers.*

Keywords: lower order h-infinity, PID controller, hurwitz criteria, robust performance, inverted pendulum

INTRODUCTION

It is well known that the majority of controllers in industry are of the proportional-integral-derivative (PID) type and lead/lag controllers of the form $C = \frac{x_1 s + x_2}{s + x_3}$ .Over the last 40 years control theory literature has been dominated by modern optimal control theory and its offshoots. These powerful techniques are based on the Youla–Jabr–Bongiorno–Kucera (YJBK) characterization of all stabilizing controllers for a given plant. However, the resulting controllers tend to be of unnecessarily high order. In fact, there are only a few results that apply to low order/fixed structure controllers. In attempting to combine the power of optimal control with low order/ fixed structure controllers one might try to obtain an analog to the YJBK parameterization. Recently, this problem has been solved for PID controllers. With the stability set parameterized, it is natural to search for a particular controller within this set based on performance and/or robustness criteria. Many such criteria can be formulated in terms of the frequency weighted H^∞ norm of a closed-loop transfer function. Using the results, it has been shown that it is possible to obtain an H^∞ optimal design using a brute force optimal search procedure for PID controllers (Ching-Ming Lee, 2004). The stability region over which the search is conducted is composed of the intersection of convex polygons. This leads to a region bounded by linear constraints. This advantage is not available in the case of first order controllers. Nevertheless, we show here that by solving sets of linear equations it is possible to obtain the complete set of stabilizing, first-order controllers which simultaneously satisfies an H^∞ constraint.

Design Preliminaries

Consider the Single Input Single Output (SISO) feedback system with the first order controller. Here we are not going for any dead time compensator for the time delayed system. The objective is to find the admissible set of X_1, X_2 and X_3 by using the polynomial stabilization method and to find the stabilizing regions.

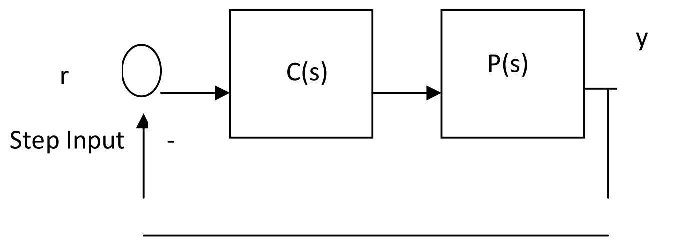

Figure 1. Feedback control system with multiplicative uncertainty

OBJECTIVES

Phase 1 – Design of Various Conventional Controller for the integral process with dead time

Phase 2 – (i) Stability and Performance analysis of the unstable systems with Mu synthesis

(ii) Design of H-Infinity Controller design.

(iii) H-Infinity PID Controller Design for the robust performance (Masami Saeki.2005 & Guillermo J. Silva. 2004)

Phase 3 – To implement the Lower Order Controller to the Real Time Inverted Pendulum

MATERIALS AND METHODS

Design Approach

The controller design part for the unstable process has been divided into two categories.

1. First order controller for the Integrating Process with Dead Time.
2. Design of Controller for the Real Time Inverted Pendulum

As a preliminary work for the controller design of real time Rotary Inverted Pendulum (RIP), we have considered the model of the RIP. The LQR controller has been calculated and implemented in LabVIEW. The obtained responses were quite satisfactory for the RIP model. In order to design and implement the Lower order robust controller /Robust PID controller for the Inverted Pendulum we need to extract the encoder output of the arm and pendulum to the external board. The encoder output will give the exact angle of the pendulum. So that we can able to calculate the error "e" by finding the difference between the 180 $^{\circ}$ and the actual angle of the pendulum, which can be used as the feedback for the controller. Since this process is under going, the real time validation of Robust PID controller will be planned in the near future (Weidong Zhang, 2002).

Need for Robust First Order Controller

In this paper, the design of lower order robust controller based on an H^{∞} performance index using polynomial stabilization has been considered. In H^{∞} controller design, the major disadvantage of the existing methods is that they lead to high-order controllers. This is the gap between theory and practice. Therefore the requirement is to design a low order controller with similar performance to the H^{∞} optimal controllers, which can find sufficiently wide use in engineering practice. We first design the H^{∞} optimal controller using Glover and Doyle's results, and obtain the corresponding performance index. Second, the desired low order controller with several parameters is chosen, e.g., a first-order controller, or a PID controller. Finally, we use the real-code genetic algorithm to find the optimal controller parameters that preserve the performance index δ. These lower order controllers finds more practical applications in the area of aircraft and space vehicle stabilizations and overcomes the disadvantages of the H^{∞} controller.

Discussion on H-Infinity based Lower Order Controller

The low order controller has many advantages such as simple hardware implementation and high reliability and is very important for the successful integration of controllers with smart structures. Designing a controller with robustness to different uncertainties in smart structure always leads to a high order controller. Alternate method of controller reduction, is to find a low order controller by reducing the full order controller. The effect of the controller reduction on the system performance is taken into account by selecting a maximum allowable controller reduction error for preserving the performance. The full order controller can be synthesized to provide optimal performance or maximum allowable controller reduction error. Linear matrix inequalities (LMIs) are utilized in those methods to design the low order controllers. The variations of structural parameters, natural frequencies and damping ratios are considered in the controller design as parametric uncertainties.

Design Problem

Consider an arbitrary LTI plant (after PADE appx) and a first order controller given by

Plant: $$P(s) = \frac{N(s)}{D(s)} = \frac{(-0.0506s) + 0.0163}{s^2 + 0.388s}$$

Controller: $$C(s) = \frac{x_1 s + x_2}{s + x_3}$$

We naturally assume that 'the plant P(s) _is stabilizable, by a controller of some order, not necessarily first order. Let us use the standard even-odd decomposition of polynomials:

$$N(s) = N_e(s^2) + sN_o(s^2) ----\rightarrow (1)$$
$$N(s) = (-0.0506s) + 0.0163$$

$$D(s) = D_e(s^2) + sD_o(s^2) ----\rightarrow (2)$$
$$D(s) = s^2 + 0.388s$$

$$\delta(j\omega) = [-\omega^2 \cdot N_o(-\omega^2)\cdot x_1 + N_e(-\omega^2)\cdot x_2 + D_e(-\omega^2)\cdot x_3 - \omega^2 \cdot D_o(-\omega^2)] + j\omega\cdot[N_e(-\omega^2)\cdot x_1 + N_o(-\omega^2)\cdot x_2 + D_o(-\omega^2)\cdot x_3 + D_e(-\omega^2)]$$

$$\delta(j\omega) = [-\omega^2(0.0506)\cdot x_1 + (-0.0163)\cdot x_2 + (-\omega^2)\cdot x_3 - \omega^2(-0.388)] + j\omega\cdot[0.0163(-\omega^2)\cdot x_1 + (-0.0506)\cdot\omega^2\cdot x_2 + 0.388(-\omega^2)\cdot x_3 + (-\omega^2)\cdot(-\omega^2)]$$

The characteristic polynomial of the closed loop system is

$$\delta(s) = D(s)(s + x_3) + N(s)(x_1 s + x_2)$$

$$= [D_e(s^2) + sD_o(s^2)](s + x_3) + [N_e(s^2) + sN_o(s)](x_1 s + x_2)$$

$$\delta(s) = [s^2\cdot D_o(s^2) + x_3\cdot D_e(s^2) + x_2\cdot N_e(s^2) + x_1\cdot s^2\cdot N_o(s^2)] + s[D_e(s) + x_3\cdot D_o(x_1) + x_2\cdot N_o(s^2) + x_1\cdot N_e(s^3)]$$

$$\delta(s) = s^2 0.388 + s^2\cdot x_3 + 0.0163\cdot x_2 + s^2(-0.0506)x_1 + s[s^2 + 0.388x_3 + (-0.0506)x_2 + 0.0163x_1]$$

$$\delta(s) = s^3 + s^2(0.388 + x_3 - 0.0506\cdot x_1) + s(0.0163\cdot x_1 - 0.0506\cdot x_2 + 0.388\cdot x_3) + 0.0163\cdot x_2$$

With s=jω

The complex root boundary is given by

$$\delta(j\omega) = 0, \qquad \omega \in (0, +\infty) ------\rightarrow (3)$$

And the real root boundary is given by

$$\delta(0) = 0, \qquad \delta_{n+1} = 0 ---------\rightarrow (4)$$

Where δ_{n+1} denotes the leading co-efficient of $\delta(s)$.

Thus, the complex root boundary is given by:

$$-\omega^2 N_o(-\omega^2)x_1 + N_e(-\omega^2)x_2 + D_e(-\omega^2)x_3 - \omega^2 D_o(-\omega^2)] = 0 ----\rightarrow (5)$$

$$-\omega^2\cdot(-0.0506)\cdot x_1 + (-0.0163)\cdot x_2 + (-\omega^2)\cdot x_3 - [(\omega^2)\cdot 0.388] = 0$$

$$\omega[N_e(-\omega^2)\cdot x_1 + N_o(-\omega^2)\cdot x_2 + D_o(-\omega^2)\cdot x_3 +^2 D_e(-\omega^2)] = 0 ----\rightarrow (6)$$

$$\omega[(0.0163)\cdot x_1 + (-0.0506)\cdot x_2 + (0.388)\cdot x_3 + (-\omega^2)] = 0$$

Note that at ω=0 equation 6 is trivially satisfied and equation 5 becomes

$$N_e(0)\cdot x_2 + D_e(0)\cdot x_3 = 0 -------------\rightarrow (7)$$

$$0.0163\cdot x_2 + (0)\cdot x_3 = 0$$

Which coincides with the condition

$$\delta(0) = 0$$

The condition δ_{n+1}=0 translates to

$$d_n + x_1\cdot n_n = 0 --------------\rightarrow (8)$$

.

Where $d_{n,} n_n$ denotes the co-efficient of s^n in D(s) and N(s) respectively.

For ω>0 we have

$$-\omega^2 N_o(-\omega^2)x_1 + N_e(-\omega^2)x_2 + D_e(-\omega^2)x_3 - \omega^2 D_o(-\omega^2)] = 0 ---\rightarrow (9)$$

$$-\omega^2\cdot(-0.0506)\cdot x_1 + (-0.0163)\cdot x_2 + (-\omega^2)\cdot x_3 - [(\omega^2)\cdot 0.388] = 0$$

$$\omega[N_e(-\omega^2)\cdot x_1 + N_o(-\omega^2)\cdot x_2 + D_o(-\omega^2)\cdot x_3 +^2 D_e(-\omega^2)] = 0 -----\rightarrow (10)$$

$$\omega[(0.0163)\cdot x_1 + (-0.0506)\cdot x_2 + (0.388)\cdot x_3 + (-\omega^2)] = 0$$

Re-write the above in matrix form:

$$\begin{bmatrix} \omega^2\cdot N_o(-\omega^2) & -N_e(-\omega^2) \\ N_e(-\omega^2) & N_o(-\omega^2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \omega^2\cdot(-0.0506) & -0.0163 \\ 0.0163 & -0.0506 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} D_e(-\omega^2)\cdot x_3 & -\omega^2 D_o(-\omega^2) \\ -D_o(-\omega^2)\cdot x_3 & -D_e(-\omega^2) \end{bmatrix} ------------\rightarrow (11)$$

$$= \begin{bmatrix} (-\omega^2)\cdot x_3 & -\omega^2(0.388) \\ -0.388\cdot x_3 & -(-\omega^2) \end{bmatrix}$$

Now consider the case when $|A(\omega)| \neq 0$ for all $\omega > 0$.

The case when $|A(\omega)| = 0$ will be discussed later.

Then

$$|A(\omega)| = \omega^2 N_o^2(-\omega^2) + N_e^2(-\omega^2) > 0$$

,

$$|A(\omega)| = \omega^2(-0.0506)^2 + (0.0163)^2$$

$$\forall\omega > 0$$

Therefore, for every x_3 the above equation in matrix form has a unique solution x_1 and x_2 at each ω>0 given by:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{|A(\omega)|} = \begin{bmatrix} N_o(-\omega^2) & N_e(-\omega^2) \\ -N_e(-\omega^2) & \omega^2 N_o(-\omega^2) \end{bmatrix}.$$

$$\begin{bmatrix} D_e(-\omega^2)x_3 - \omega^2 D_o(-\omega^2) \\ D_o(-\omega^2)x_3 - D_e(-\omega^2) \end{bmatrix} -----\rightarrow (12)$$

In other words

$$x_1(\omega)=\frac{1}{|A(\omega)|}\left(\left[N_o(-\omega^2)D_e(-\omega^2)-N_e(-\omega^2)D_o(-\omega^2)\right]x_3-\omega^2N_o(-\omega^2)D_o(-\omega^2)-N_e(-\omega^2)D_e(-\omega^2)\right)$$

$$x_1(\omega)=\frac{(0.0506(\omega^2)-0.0063244)\cdot x_3+0.0359328(\omega^2)}{0.00256036(\omega^2)+0.00026569}$$

$$x_2(\omega)=\frac{1}{|A(\omega)|}\left(\left[-N_e(-\omega^2)D_e(-\omega^2)-\omega^2N_o(-\omega^2)D_o(-\omega^2)\right]x_3+\omega^2N_e(-\omega^2)D_o(-\omega^2)-\omega^2N_o(-\omega^2)D_e(-\omega^2)\right)$$

$$x_2(\omega)=\frac{(0.0359328\cdot\omega^2+0.0063244\cdot\omega^2)x_3-(0.0506\cdot\omega^4)}{(0.00256036\cdot\omega^2)+0.00026569}$$

For a fixed value of x_3, let ω run from 0 to ∞. The above equations trace out a curve in the x_1 - x_2 plane corresponding to the complex root boundary. These curves along with the straight lines equation (7) and equation (8) partition the parameter space into a set of open root distribution invariant regions.

If the possibility $|A(\omega)|=0$ is considered for some $\omega\neq 0$. The assumption of stabilisability of the plant rules out this possibility.

Let

$$|A(\omega)| = \omega^2 N_o^2(\omega^2) + N_e^2(-\omega^2) = 0, ---\rightarrow (13)$$

For some $\omega\neq 0$. Since $N_o^2(\omega^2), N_e^2(\omega^2)\geq 0$, equation (13) holds if and only if

$$N_o^2(\omega^2) = N_e^2(-\omega^2) = 0. -------\rightarrow (14)$$

From equation (11) it follows that

$$D_e(-\omega^2)x_3 - \omega^2 D_o(-\omega^2) = 0,$$
$$-D_o(-\omega^2)x_3 - D_e(-\omega^2) = 0,$$
$$therefore$$
$$\omega^2 D_o(-\omega^2) + D_e^2(-\omega^2) = 0 ------\rightarrow (15)$$
$$since\ D_o(\omega^2), D_e(-\omega^2)\geq 0,$$

Equation (15) holds good if and only if $D_o(\omega^2) = D_e(-\omega^2) = 0 -------\rightarrow (16)$

From equation (14) and (16), it follows that equation (13) has a solution for ω≠0 if and only if D(s) and N(s) have a common factor $s^2+\omega^2$. Therefore, the case $|A(\omega)|$=0 for some ω need not be considered.

RESULTS AND DISCUSSIONS

After the peer analysis and mathematical process, the stabilizing layers of X_1, X_2 and X_3 has been plotted and the stability test through Routh Hurwitz criteria was also carried out for the particular values.

Table-1. Set of X_1, X_2 and X_3 values which stabilizes the δ

S.No	X_1	X_2	X_3
1.	7.5	11	2.4
2.	7.5	29	2.4
3.	7.5	27	2.4
4.	7.5	25	2.4
5.	7.5	22	2.4
6.	7.5	20	2.4
7.	7.5	19	2.4
8.	7.5	17	2.4
9.	7.5	16	2.4
10.	7.5	15	2.4
11.	7.5	10	2.4

Table.2. Checked set of X_1, X_2 and X_3

X_1	X_2	X_3
7.5	11	2.4

Table.3. Routh Table for δ

Routh Hurwitz 1st column is	
s^3	1
s^2	2.40850
s^1	0.4224053249
s^0	0.1793

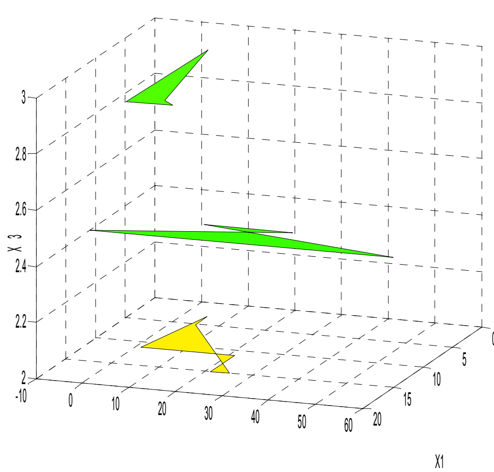

Figure 2. Admissible set of X_1, X_2 and X_3 values for First order Controller

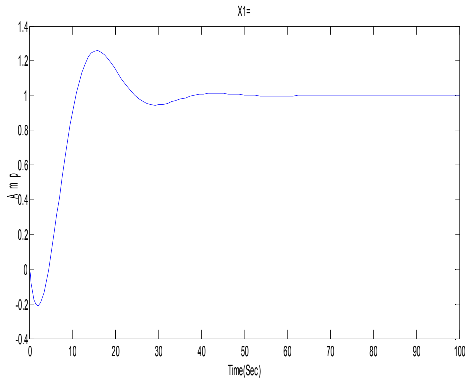

Figure 3. Step response for the X_1, X_2 and X_3 values as in table

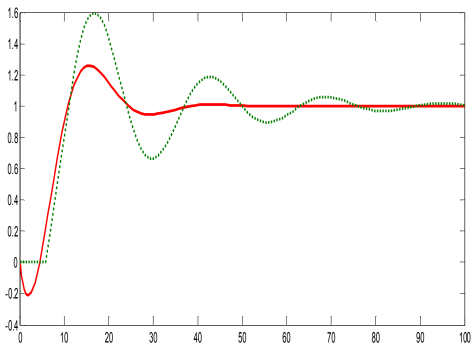

Figure 4.Step response for the appx and time delayed plant.

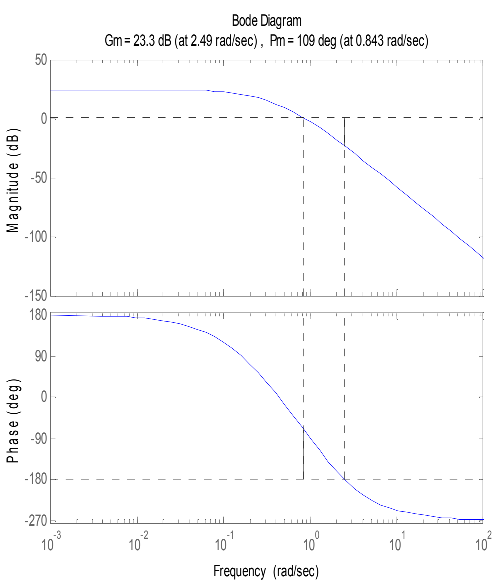

Figure 5. Bode plot for the PC

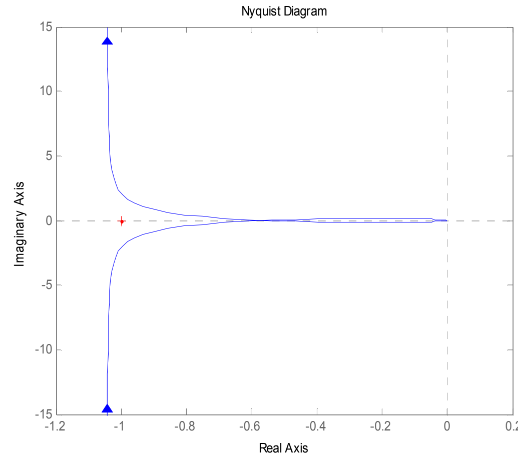

Figure 6. Nyquist plot of closed loop system(PC)

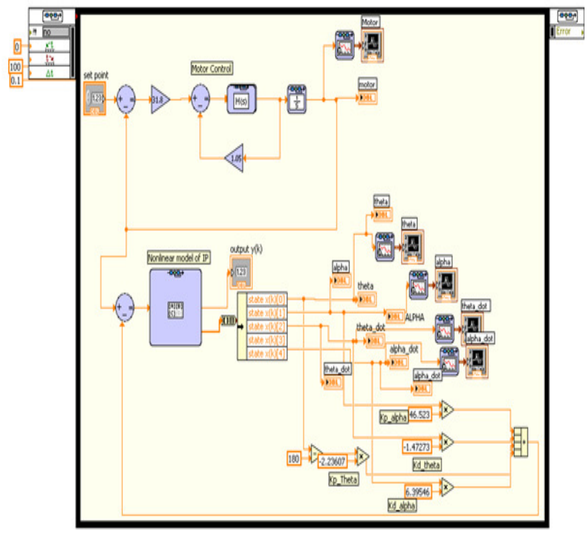

Figure 7. Implementation of LQR controller for the RIP model in LabVIEW

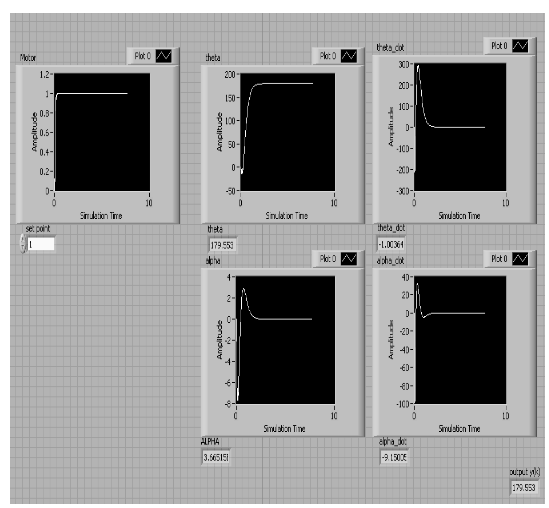

Figure 8. Simulation results of RIP with the all four states α, α ,θ, θ in LabVIEW

FUTURE WORKS

1) To retune the controller for the RIP-MIMO transfer function.
2) To validate the real time response with the various controller tuning.

CONCLUSIONS

The results of first order robust controller for the time delayed system has been reported in this paper. Layers of stabilizing values of X_1 and X_2 for the various fixed values of X_3 has been plotted as shown in Fig.1 and the stability of the controller with respect to plant were also analyzed as showed in Fig.5 and Fig.6. As a part of real time validation using Rotary Inverted Pendulum (RIP), the LQR controller has been designed and simulated for the model of the RIP, also it results in the satisfactory simulation results as shown in the Fig.7. and Fig.8.

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