Electrohydrodynamic Kelvin-Helmholtz Instability in a Fluid Layer Bounded above by a Porous Layer and below by a Rigid Surface

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Abstract
The surface instability of Kelvin-Helmholtz type bounded above by a porous layer and below by a rigid surface is investigated using linear stability analysis. Here we adopt the theory based on electrohydrodynamic as well as Stokes and lubrication approximations. We replace the effect of boundary layer with Beavers and Joseph slip condition. Here we have studied the effect of electric field on KHI in a fluid layer bounded above by a porous layer and below by a rigid surface. The dispersion relation is obtained through normal mode technique using suitable boundary and surface conditions and results are depicted graphically. From this it is clear that the effect of electric field with porous layer is more effective than the effect of compressibility in reducing the growth rate of RTI. Also, these results shows that with a proper choice of electric parameter it is possible to control the growth rate of EKHI and also it is found that the electric field stabilizes the system

Key words: *Electrohydrodynamics, KHI, BJ slip condition, porous layer*

1. INTRODUCTION

Kelvin-Helmholtz Instability (KHI) after Lord Kelvin (1910) and Hermann von Helmholtz (1868) can occur when velocity shear is present within a continuous fluid or when there is sufficient velocity difference between two fluids. One example is wind blowing over a water surface where the wind causes the relative motion between the stratified layers (i.e., water and air). The instability will manifest itself in the form of waves being generated on the water surface. The theory can be used to predict the onset of instability and transition to turbulent flow in fluid different densities moving at various speeds. The KHI of a Newtonian incompressible fluid in a composite layer bounded by a densely packed fluid saturated porous layer on one side and another side by an impermeable rigid surface is investigated in this paper. This can occur when velocity shear is present at the interface within a continuous fluid or when there is sufficient velocity difference across the interface between two fluids. Helmholtz (1868) studied the dynamics of two fluids of different densities when a small disturbance such as a wave is introduced at the boundary connecting the fluids. If surface tension can be ignored, and for some short enough wavelengths, two fluids in parallel motion with different velocities and densities yielded an interface that is unstable for all speeds. The existence of surface tension stabilizes the short wavelength instability however, and theory then predicts stability until a velocity threshold is reached. The theory with surface tension included broadly predicts the onset of wave-formation in the important case of wind-over-water. Also the study of this instability becomes applicable to inertial confinement fusion and plasma–Beryllium interface. The KHI in a composite layer differs from the KHI in two fluid layers. The KHI arises when two uniform fluids, separated by a horizontal boundary, are in relative motion. Because of its relevance to astrophysical, geophysical and laboratory situations, this problem has been analyzed by several authors (Shore, 1992, Chandrasekhar, 1961). Without surface tension, this streaming is unstable no matter how small the velocity difference between the layers may be. It was shown by Kelvin (1910) that the surface tension will suppress the instability if the difference in velocity is sufficiently small. From an industrial view point, the momentum transfer and the KHI in composite region provides impetus for effective design of porous bearings in lubrication process, particularly in the slider bearings and in the effective design of target in inertial fusion energy (IFE). Because of these importances the KHI is investigated in this paper using the linear stability analysis. Chandrasekhar (1961) gave an introduction to classical KHI. He discussed the effects of surface tension, variable density, streaming velocity, rotation and applications of magnetic field on the instability behaviour. The experimental observation of KHI has been given by Francis (1954). The effect of rotation and a general oblique magnetic field on the KHI has been studied by Sharma and Srivastava (1968). The study of electrohydrodynamic (EHD) KHI of free surface charges, separating two semi-infinite dielectric fluids and influenced by an electric field, has been discussed by Elshehawey and Mahmoud (1984). The main difference between KH and RT instabilities in inclusion of $(V.\nabla)V$, which is a non-linear term in the perturbation equations. Storesletten (1982) has studied the linear stability of stratified horizontal flows of an inviscid compressible fluid. He has shown that a shear in the horizontal directions always gives rise to instabilities and also that all shear flows are unstable if the external force field vanishes. The same results are also obtained for homogeneous incompressible fluids by Ellingsen and Palm (1975) and Landehl (1980).

The mechanical stability of the interfaces in immiscible fluid-fluid systems is a subject of increasing interest. Theoretical models were developed to take into account more accurately the physical and chemical properties of interfaces. Fluid-fluid interfaces can also exhibit electrical charges. External constraints can be imposed, for instance external electric charges, a non-equilibrium density of charges, or an imposed electric current (Haus and Melcher (1989)). Scattered literature exists, dating back to the nineteenth century, on a variety of electrofluid phenomena. However, only recently the term "electrohydrodynamics" has come into usage to describe electrofluid interactions between two fluids (Melcher (1981)). Electrohydrodynamics (EHD) studies the interplay of mechanical and electrical forces in fluids. In a first approximation it is assumed that the electrical currents are very weak and therefore magnetic effects are negligible. Maxwell's equations are then reduced to Gauss's law and the charge conservation law. Within the past few years, a number of papers have appeared on studies of surface stability in the presence of electric fields(El-Dib and Moatimid(1994), Elsheawey & Mahmed (1984)).

Only few trials have been made to study the effect of electric field on the stability of rotating fluids. For example, El-Dib and Moatimid (1994) studied the effect of a periodic rotation and uniform axial and perpendicular electric fields on the stability of two dielectric inviscid fluids separated by a cylindrical interface using a multiple time scales. Mohamed et al.(1985) studied the electrohydrodynamic stability of a rotating dielectric jet bounded by an infinite dielectric rotating fluid. Takashima(1976) considered the effect of uniform rotation on the onset of convective instability in a dielectric fluid confined between two horizontal planes under the simultaneous action of a vertical ac electric field and a vertical temperature gradient. Oliveri et al.,(1987) examined the effect of rotation on the EHD stability of a plane layer of fluid subjected to unipolar injection of charge.

Elhefnawy (1992) studied the non-linear KHI problem under the influence of an oblique electric field by employing the method of multiple scales. He combined the cases of normal and tangential fields. He found that the nonlinear effects may be stabilized or destabilized depending on both density and dielectric constant. The importance of the KHI problem has been demonstrated by Benjamin and Bridges (1997) who have given an excellent reappraisal of the classic KHI problem in hydrodynamics. They have shown that the problem admits of a canonical Hamiltonian formulation and obtained several new results. More recently Sharma and Kumar (1998) have studied the RTI of two superposed conducting Walter's B' electroviscoelastic fluids in hydromagnetics while Allah (1998) has investigated the effects of magnetic field and heat and mass transfer on the KHI of superposed fluids.

In this study the flow in the porous layer is governed by the Darcy equation and that in a thin fluid film is governed by Navier-Stokes equation. Following Babchin et al., (1983) and Rudraiah et al.,(1997), a simple theory based on Stokes and lubrication approximations is used in this study by replacing the effect of the boundary layer with a Beavers and Joseph(BJ, 1967) slip condition, with the primary objective of using porous layer is to suppress the growth rate of KHI. In the above studies the fluid has been considered to be Newtonian. To the best of my knowledge, the effect of electric field on the Kelvin-Helmholtz instability problem has not been investigated yet.

2. Mathematical Formulation

The physical configuration is shown in Fig.1. We consider a thin target shell in the form of a thin film of unperturbed thickness *h* (Region 1) filled with an incompressible, viscous, poorly electrically conducting light fluid of density ρ_f

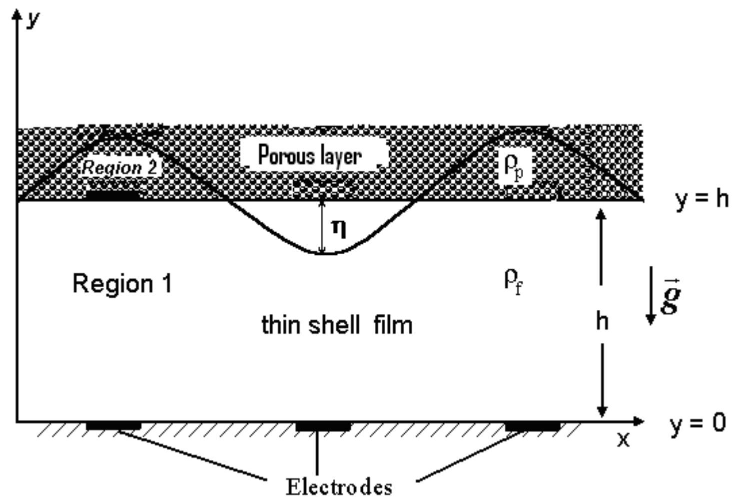

Fig. 1: Physical Configuration

bounded below by a rigid surface at *y=0* and above by an incompressible, viscous poorly conducting heavy fluid of density ρ_p saturating a dense nanostructured porous layer of large extent compared to the shell thickness *h*. Also, the electrodes are embedded at the rigid surface y = 0 as well as at the interface $y = h$ and there by an electric field is generated in fluid-porous medium composite system in addition with transverse magnetic field. The fluid in the thin film is set in motion by acceleration normal to the interface. The small perturbations are amplified when acceleration is directed from the lighter fluid in the thin film to the heavier fluid in the porous lining. KHI can occur when there is sufficient velocity difference across the interface between two fluids. To investigate this KHI, we consider a rectangular coordinate system *(x, y)* with the x-axis parallel to the film and y-axis normal to it. The perturbed interface $\eta(x,t)$ is along the y direction.

The basic equations for clear fluid layer (region 1) and those for porous layer (region 2) are as given below:

Region-1:

The conservation of mass for an incompressible fluid

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

The conservation of momentum

$$\rho_f \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p + \mu_f \nabla^2 \vec{q} + \rho_e E \tag{2.2}$$

The conservation of electric charges

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla)\rho_e + \nabla \cdot \vec{J} = 0 \tag{2.3}$$

The Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_e} \tag{2.4a}$$

$$\nabla \times \vec{E} = 0 \Rightarrow \vec{E} = -\nabla \phi \tag{2.4b}$$

$$\vec{J} = \sigma \vec{E} \tag{2.4c}$$

$$\sigma = \sigma_0 \left[1 + \alpha_h (C - C_0) \right]. \tag{2.4d}$$

Region-2:

$$Q = -\frac{k}{\mu} \frac{\partial p}{\partial x} \tag{2.5}$$

where $\vec{q} = (u, v)$ the fluid velocity, $\vec{E}$ the electric field, $\vec{H}$ the magnetic field, $\vec{J}$ the current density, σ the electrical conductivity, ρ_e the density of charges, ε_e the dielectric constant, k the permeability of the porous medium, ϕ the electric potential, p the pressure, C the concentration, μ_0 magnetic permeability, σ_0 the electrical conductivity at the reference concentration C_0, $\vec{Q} = (Q, 0, 0)$ the uniform Darcy velocity, α_h is the volumetric expansion coefficient of σ, μ the fluid viscosity and ρ the fluid density.

The electrical conductivity σ varies with concentration C of DT as in Eq.(2.4d). Then assuming negligible advection of concentration, we have

$$\frac{d^2C}{dy^2} = 0 \tag{2.4f}$$

with

$$C = C_0 \quad \text{at} \quad y = 0 \tag{2.4g}$$
$$C = C_1 \quad \text{at} \quad y = h. \tag{2.4h}$$

Solving Eq. (2.4f) using the above conditions and substituting the solution so obtained in Eq.(2.4d), we get

$$\sigma = \sigma_0 \left(1 + \alpha y \right) \approx \sigma_0 e^{\alpha y} \quad (\because \alpha << 1) \tag{2.4i}$$

where $\alpha = \alpha_h \Delta C$ and $\Delta C = C_1 - C_0$. We assume the frequency of charge distribution is smaller than the corresponding relaxation frequency of the electric field, and hence the time derivative of ρ_e is negligible compared to $\nabla \cdot (\sigma E)$ in Eq.(2.3). From this, we get

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \alpha\,\frac{\partial\phi}{\partial y} = 0. \tag{2.5}$$

The above equation has to be solved subject to the boundary conditions

$$\phi = v_0\,\frac{x}{h} \qquad \text{at} \quad y = 0 \tag{2.6a}$$

$$\phi = v_0\,\frac{(x - x_0)}{h} \quad \text{at} \quad y = h\,. \tag{2.6b}$$

where v_0 is the applied electric potential. These conditions arise due to embedded electrodes at $y = 0$ and $y = h$ and permits a linear variation of ϕ with x.

The basic equations are simplified using the following Stokes and lubrication and electrohydrodynamic approximations (See Rudraiah et al, 1997):

(i) The electrical conductivity of the liquid, σ, is negligibly small, i.e., $\sigma << 1$.

(ii) The film thickness h is much smaller than the thickness H of the dense fluid above the film. That is $h << H$

(iii) The surface elevation η is assumed to be small compared to film thickness h. That is $\eta << h$

(iv) The Strauhal number S, a measure of the local acceleration to inertial acceleration in Eq.(2.2), is negligibly small.

That is

$$S = \frac{L}{T\,U} << 1$$

where $U = \nu / L$ is the characteristic velocity, ν the kinematic viscosity, $L = \sqrt{\gamma / \delta}$ the characteristic length and $T = \mu\gamma / h^3\delta^2$ the characteristic time.

Under these approximations Eqs.(2.1) and (2.2) for fluid in the film, after making dimensionless using

$$u^* = \frac{u}{\delta h^2 / \mu_f}, v^* = \frac{v}{\delta h^2 / \mu_f}, p^* = \frac{p}{\delta h}, \rho_e^* = \frac{\rho_e h^2}{\varepsilon_e v_0}, E^* = \frac{Eh}{v_0}, Q^* = \frac{Q}{\delta h^2 / \mu_f},$$
$$t^* = \frac{t}{\delta h / \mu_f}, x^* = \frac{x}{h}, y^* = \frac{y}{h} \tag{2.7}$$

become (after neglecting the asterisks for simplicity)

Region 1:

$$0 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \tag{2.8}$$

$$0 = \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + We\rho_e E_x \tag{2.9}$$

$$0 = \frac{\partial p}{\partial y} + We\rho_e E_y \tag{2.10}$$

where $We = \varepsilon_e v_0^2 / \delta h^3$ is the electric number.

Region 2:

$$Q = -\frac{1}{\sigma_p^2}\frac{\partial p}{\partial x} \tag{2.11}$$

where

$\sigma_p = h / \sqrt{k}$ is the porous parameter.

Substituting the Eq.(2.7) in Eq.(2.5) and in the boundary conditions (2.6), we obtain

$$\frac{\partial^2\phi}{\partial y^2}+\alpha\frac{\partial\phi}{\partial y}=0 \tag{2.12}$$

with boundary conditions

$$\phi = x \quad \text{at} \quad y=0 \tag{2.13}$$

$$\phi = x - x_0 \quad \text{at} \quad y=1. \tag{2.14}$$

The solution of Eq.(2.12), using the above boundary conditions, is

$$\phi = x - \frac{x_0}{1-e^{-\alpha}}\left(1-e^{-\alpha y}\right). \tag{2.15}$$

Equation (2.4a), using Eq.(2.15), becomes

$$\rho_e = -\frac{x_0\alpha^2}{1-e^{-\alpha}}e^{-\alpha y} \tag{2.16}$$

and hence

$$\rho_e E_x = -\rho_e\frac{\partial\phi}{\partial x} = \frac{\alpha^2 x_0 e^{-\alpha y}}{(1-e^{-\alpha})}. \tag{2.17}$$

3. Dispersion Relation

To find the dispersion relation, first we have to find the velocity distribution from Eq. (2.9) using the following boundary and surface conditions:

$$u = 0 \quad \text{at} \quad y = 0 \tag{3.1}$$

$$\frac{\partial u}{\partial y} = -\alpha_p\sigma_p(u_B - Q) \quad \text{at} \quad y = 1 \tag{3.2}$$

where

$$u = u_B \quad at \quad y=1$$

$$v = \frac{\partial\eta}{\partial t} \quad \text{at} \quad y = 1 \tag{3.3}$$

$$p = -(1\pm We)\eta - \frac{1}{B}\frac{\partial^2\eta}{\partial x^2} \quad \text{at} \quad y = 1. \tag{3.4}$$

Here $B=\delta h^2/\gamma$ is the Bond number and $\eta=\eta(x,y,t)$ is the elevation of the interface.

The solution of (2.9) subject to the above conditions is

$$u = P\frac{y^2}{2} - Wea_3e^{-\alpha y} + a_1y + a_2 \tag{3.5}$$

where

$$a_3 = \frac{x_0}{1-e^{-\alpha}}$$

$$a_2 = Wea_3$$

$$a_1 = -\frac{1}{(1+\alpha_p\sigma_p)}\left[P\left(1+\frac{\alpha_p\sigma_p}{2}-\frac{1}{\sigma_p^2}\right)+Wea_3(\alpha e^{-\alpha}+\alpha_p\sigma_p(1-e^{-\alpha}))\right]$$

$$p = \frac{\partial p}{\partial x}.$$

After integrating Eq.(2.8) with respect to *y* between *y* = 0 and 1 and using Eq.(3.5), we get

$$v(1) = \left[(1\pm We)\frac{\partial^2\eta}{\partial x^2}+\frac{1}{B}\frac{\partial^4\eta}{\partial x^4}\right]\Delta \tag{3.6}$$

where

$$\Delta=\frac{1}{\alpha_p^2(1+\alpha_p\sigma_p)}-\frac{5+2\alpha_p\sigma_p}{6(1+\alpha_p\sigma_p)}.$$

Then Eq.(3.3), using Eqs.(3.6) and (3.4), becomes

$$\frac{\partial\eta}{\partial t}=\left[(1\pm We)\frac{\partial^2\eta}{\partial x^2}+\frac{1}{B}\frac{\partial^4\eta}{\partial x^4}\right]\Delta\ . \qquad (3.7)$$

To investigate the growth rate, *n*, of the periodic perturbation of the interface, we look for solution of Eq.(3.7) in the form

$$\eta=\eta(y)\exp\{i\ell x+nt\} \qquad (3.8)$$

where ℓ is the wave number and $\eta(y)$ is the amplitude of perturbation of the interface.

Substituting Eq.(3.8) into (3.7), we obtain the dispersion relation in the form

$$n=\ell^2\left[(1\pm We)-\frac{\ell^2}{B}\right]\Delta. \qquad (3.9)$$

The positive or negative sign in the second term in Eq.(3.9) will depends on whether voltage difference is applied in the same direction of the gravity or opposite direction. So that the Eq.(3.9) takes the form

$$n=\ell^2\left[(1-We)-\frac{\ell^2}{B}\right]\Delta. \qquad (3.10)$$

Also, Eq. (3.10) can be expressed as

$$n=n_b-\ell\beta v_a \qquad (3.11)$$

where

$$n_b=\frac{\ell^2}{3}\left[1-\frac{\ell^2}{B}\right],$$

$$\beta=\Delta\ell\left[(1-We)-\frac{\ell^2}{B}\right],$$

$$v_a=\left(\frac{1-3\Delta}{3\Delta}\right)\left(1-\frac{\ell^2}{B}\right)+\frac{We}{\left((1-We)-\frac{\ell^2}{B}\right)}.$$

4. RESULTS AND DISCUSSION

In this study we have shown the surface instability of KH type in a fluid layer bounded above by a porous layer and below by a rigid surface is affected by the effect of electric field. The growth rate *n* given by the relation (3.10) is numerically computed for different values *We*, *B* and σ_p. The results are depicted in the figures 2 to 4 of growth rate *n* versus the wave number ℓ.

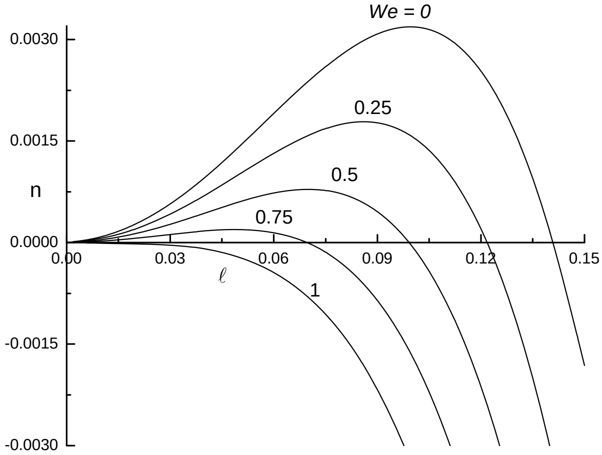

Fig. 2: Growth rate, *n* versus the wavenumber, ℓ for different values of electric parameter *We* when $\alpha_p = 0.1$, $\sigma_p = 4$ and $B = 0.02$.

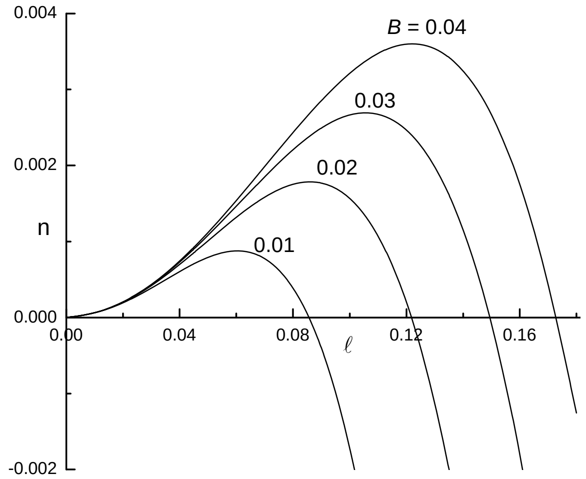

Fig. 3: Growth rate, *n* versus the wavenumber,
ℓ for different values of Bond number *B* when $\alpha_p = 0.1$, $\sigma_p = 4$ and $We = 0.25$.

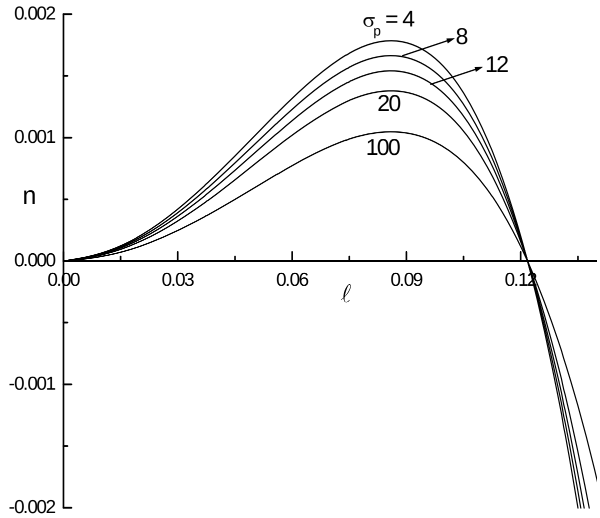

Fig. 4: Growth rate, *n* versus the wavenumber,
ℓ for different values of porous parameter σ_p when $\alpha_p = 0.1$, $B = 0.02$ and $We = 0.25$.

From Fig.2 we found that the growth rate decreases with increase in the values of *We* = 0, 0.25, 0.5, 0.75 and 1. It is clear that the growth rate decreases with increase in the electric energy compared to in absence of electric field. From Fig.3, it is observe that the growth rate increases with increase in the Bond number in the range of 0.04 to 0.01. The Bond number *B* being the reciprocal of surface tension implies that an increase in surface tension decreases the growth rate and hence makes the interface more stable. Finally, from Fig.4 we found that as σ_p increases from 4 to 100 the growth rate decreases and move towards neutral stability. We conclude that an increase in σ_p also stabilizes the EKHI due to the resistance offered by the solid particles of the porous layer to the fluid.

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